Collision/Contact Models for the Dynamic Simulation of Complex Environments

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Abstract

In this paper we examine the constraint equations developed for two general dynamic simulation systems from a new perspective. A contact space framework, similar to the operational space framework used for robotic control, is developed which allows the constraint equations for contact and collision to be easily specified. This framework also allows us to scrutinize the multiple relations that exist between the dynamic models used in simulation and the models originally developed for robotic control and analysis. This framework has been used to develop a simulator that can model complex interaction between generalized articulated mechanical systems at interactive rates.

1 Introduction

In recent years, there have been many efforts to accurately simulate physical environments in both robotics and computer graphics. A physically accurate simulation can be used to obtain insight into the real-world behavior of a robotic, fabrication or other dynamic environment. Two recent simulator systems have shown that it is possible to simulate a large number of reasonably complicated environments at interactive speeds. Both these simulators are often described as "analytical" methods because they solve numerically for the exact contact forces and impulses required to guarantee that the simulated bodies never inter-penetrate. These methods differ from "penalty" methods that introduce restoring forces, typically proportional to the amount of penetration, only after the objects have penetrated.

The simulator *Coriolis* [1] [2] [3], developed by David Baraff constructs a system of simultaneous constraints, which is solved to find the exact contact forces and impulses required to prevent inter-penetration. The simulator *Impulse* developed by Brian Mirtich and John Canny [8] models all inter-object interaction as a series of tiny micro-collisions. Only impulses (not contact forces) are applied to the objects to model contact. Despite this modeling simplification, in tests comparing simulated and real systems, *Impulse* has demonstrated that it can accurately model the macroscopic behavior of the physical system. In addition *Impulse* takes into account the effect of friction during impact which is not considered in [1].

Both of these systems were originally developed for simulating the motion of simple rigid bodies. While *Impulse* has been extended to support generalized articulated bodies [9] the constraint equations are still fundamentally based on the equations of motion for simple (un-articulated) rigid body systems. In the resulting equations most the underlying meaning of the constraints is no longer discernible.

Figure 1: An Interactive Dynamic Environment

In our work we have focused on the development of simulation systems for generalized articulated body systems. An example is illustrated in figure 1. We derive the contact constraint equations using modeling methodologies and principles from robotics to obtain equations that are easier to understand and highlight the connections between dynamic constrained motion simulation and physical models for robotic control. In this paper we discuss how the contact constraint equations given by Baraff and Mirtich can be obtained for generalized articulated systems and discuss their connection with many robotic control concepts.

The rest of the paper is organized as follows. In Section 2 we introduce some notation and describe the motion of unconstrained generalized articulated systems. In Section 3 we introduce the *contact space* dynamic equations. This space is very similar to the *operational space* formulation common in robotic control [14]. In Section 4 we describe how the collision penetration constraints introduced by Baraff [1] can be defined for generalized articulated systems. We also show the strong connection these constraint equations share with the formulation for control of redundant systems. In Section 5 we look briefly at the differences between Mirtich's and Baraff's models of impact and discusses how these differences affect the contact space constraints. Section 6 describes the contact force constraints needed when objects are in resting or sliding contact with one another. Some extensions to the contact space model are discussed in Section 7. Lastly we conclude with a brief description of the simulator *simpact* we have developed. *Simpact* makes use of the contact space formulation to simulate interaction between generalized objects.

Note that a complete description of the collision/contact equations is beyond the scope of this paper and a basic familiarity with Baraff's [1] and Mirtich's [8] simulation systems is assumed.

2 Free Space Motion

The configuration of an n-DOF (degree-of-freedom) object can be described by q, a set of n independent generalized coordinates. Joint angles are generally used as the generalized coordinates for robots, but other sets of parameters that satisfy the above constraints are possible. The equation of motion in terms of generalized coordinates is given by

$$\Gamma = A(q)\ddot{q} + b(q,\dot{q}) + g(q) \tag{1}$$

where $A(q)$ designates the kinetic energy matrix, $b(q,\dot{q})$ the centrifugal and Coriolis force vector, $g(q)$ the gravity force vector, and Γ the generalized torque vector of the object. There currently exist many methods for computing these equations [13] [11] [6] [5].

To dynamically simulate an object using its equations of motion, Eq. 1 is solved for $\ddot{q}$.

$$\ddot{q} = A^{-1}(q)[\Gamma - b(q,\dot{q}) - g(q)] \tag{2}$$

Methods exist for computing A^{-1} directly without computing A and then inverting [6]. These methods are in general more computationally expensive for systems with a small number of degrees of freedom, but have better asymptotic performance and are therefore more appropriate when the DOF of the system becomes large. Once the configuration space acceleration of an object is known, an ordinary differential equation (ODE) solver can be used to numerically integrate the equations of motion from a initial starting configuration and velocity.

When two or more objects exist in an environment their configuration vectors can be combined into a new configuration vector $q = [q_1^T\, q_2^T \ldots q_n^T]^T$ where n is the number of independent objects in the environment and $q_i, 1 \leq i \leq n$, is the configuration vector for the ith object. Analogously the equations of motion of each object can be stacked to yield the equations of motion for the entire system.

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix}; A = \begin{bmatrix} A_1(q_1) & & & \\ & A_2(q_2) & & \\ & & \ddots & \\ & & & A_n(q_n) \end{bmatrix};$$

$$b = \begin{bmatrix} b_1(q_1,\dot{q}_1) \\ b_2(q_2,\dot{q}_2) \\ \vdots \\ b_n(q_2,\dot{q}_n) \end{bmatrix}; g = \begin{bmatrix} g_1(q_1) \\ g_2(q_2) \\ \vdots \\ g_n(q_n). \end{bmatrix} \tag{3}$$

The equations of motion for a entire set of objects can therefore be treated as one large system and equation 2 used to find the configuration space acceleration $\ddot{q}$ for the entire system.

When contact or collision occurs between objects in the system one or more of the terms of q is no longer independent. In this case constraint forces or impulses must be applied at the contact points in order to prevent inter-penetrations between the objects.

3 Contact Space

When a collision between two bodies A, B occurs there exists a set of points c_a on body A and c_b on body b such that at collision time t, $c_a(t) = c_b(t)$. If the objects in the world are modeled as being constructed from the union of convex polyhedron, the contact region between two bodies will be defined by a set of convex polygons. In this situation it is sufficient to consider contact only at the extremal-points of the contact region created by the intersection of the contact surfaces [18].

Given this polygonal contact region assumption, only a finite number of contact points need to be considered. Let m be the number of contact points; for $1 \leq i \leq m$ we shall define c_{a_i} as the i^{th} contact point attached to body A. In addition we can associate with each contact point a unit normal n_i perpendicular to the contact region. With this information we can define a set of contact parameters x_i that describe the relative distance between contact point c_{a_i} and c_{b_i} in the direction perpendicular to the contact surface. This parameter can be written as

$$x_i = n_i^T(c_{a_i} - c_{b_i}), \tag{4}$$

for $1 \leq i \leq m$. These "contact" parameters $x = [x_1 x_2 \ldots x_m]^T$ form a space that describes the motion of the bodies during contact and collision. An example of this space for two contacts is illustrated in Figure 2. Although, for clarity, this space is shown when the objects are separating, this space is only defined when the objects are in contact (i.e. $x = \vec{0}$). While $x = \vec{0}$, other properties of this space, such as the velocity $v = \dot{x}$ and acceleration $a = \ddot{x}$, may be non-zero. As we will see below, these terms will become useful when modeling the interaction between the bodies in the system.

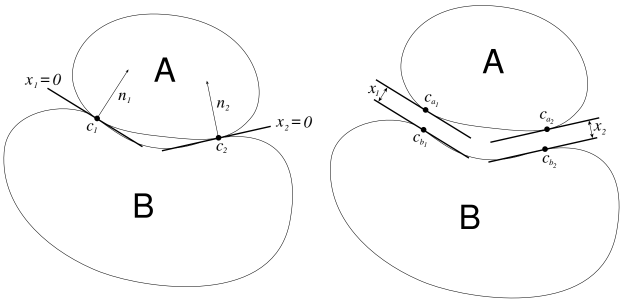

Figure 2: Contact space parameter $x = [x_1, x_2]$ locally describe the motion at the contact normal to the surface.

The velocity v_{a_i} of c_{a_i} is given by the equation

$$v_{a_i} = v_a + \omega_a \times r_{a_i}, \tag{5}$$

where v_a and ω_a are the linear and rotational velocity vectors for body A. The vector r_{a_i} is the displacement vector between c_{a_i} and the reference frame of body A. Equation 5 can be written in matrix form as

$$v_{a_i} = [I \quad -\hat{r}_{a_i}] \, J_a \dot{q}, \tag{6}$$

where I is the 3×3 identity matrix, $\hat{r}_{a_i}$ is the skew-symmetric matrix

$$\hat{r}_{a_i} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

and J_a is the Jacobian for body A, such that

$$\begin{bmatrix} v_a \\ \omega_a \end{bmatrix} = J_a \dot{q}.$$

Projecting this velocity along the contact surface normal n_i we can obtain an expression for the contact space velocity $\dot{x}_i = v_i$,

$$v_i = {n_i}^T(v_{a_i} - v_{b_i}). \tag{7}$$

Combining equations 6 and 7 we can construct a $1 \times n$ contact point Jacobian such that $v_i = J_{c_i}\dot{q}$. This Jacobian is given by the equation

$$J_{c_i} = {n_i}^T \, [I \quad -\hat{r}_{a_i}] \, J_a - {n_i}^T \, [I \quad -\hat{r}_{b_i}] \, J_b \tag{8}$$

The Jacobians for each contact can be stacked together to form a Jacobian J_c that describes the relative velocities perpendicular to the contact surface between all the contact points in the system

$$J_c = \begin{bmatrix} J_{c_1} \\ J_{c_2} \\ \vdots \\ J_{c_m} \end{bmatrix}. \tag{9}$$

The contact space velocity can now be written compactly as

$$v = J_c \dot{q}. \tag{10}$$

Note that the matrix J_c may not necessarily be of full rank as some of the rows of J_c may be redundant. Such a situation occurs for example when a four legged table rests on a flat surface. At least one of the contact points velocity is a linear combination of the other three.

With this definition it is now possible to express the operational (contact) inertia matrix for this space

$$\Lambda^{-1} = J_c A^{-1} {J_c}^T. \tag{11}$$

This matrix is very similar to the operational space inertia matrix used in robotic control [14]. The matrix Λ represents the effective mass seen at the contact points. The diagonal terms Λ_{ii}^{-1} tell us the rate at which the two contacting bodies will recede from each other if a unit force is applied at the contact point (in the direction n_i on body A and $-n_i$ on body B). The off diagonal terms tell us about the coupling in the system. A unit force at contact point c_i will cause an acceleration at contact point c_j of Λ_{ij}^{-1}. Since Λ^{-1} is positive semi-definite and hence symmetric, the force at contact point c_j would cause an equal acceleration on c_i. The contact space inertia matrix has many useful properties which we will make use of in our derivation below.

4 Impulse Force Resolution

When $v_i < 0$ for some element in v, two or more bodies in the system are are colliding. In a system of rigid bodies an impulse must be applied to prevent the objects from inter-penetrating. Several analytical methods have been proposed to compute the needed impulse forces. Propagation methods model multiple collisions as a series of independent events. Each collision is applied one at a time until $v_i \geq 0$ for all contact points c_i[1]. Simultaneous methods solve for and apply impulse forces simultaneously at all contact points at the same time. Propagation methods are simpler to implement since they consider the effect of only one contact point at a given time, however, in some situations a large number of iterations are required before all the collisions are resolved. In contrast, simultaneous models will produce a non-colliding solution in only one iteration. Because a independent collision is a special case of a simultaneous collision we will look at the simultaneous solution first and then consider the simplifications possible when only one contact is considered.

Baraff proposed an empirical model to calculate collision impulses for simultaneous collisions. In this model three constraints must be satisfied when a collision occurs.

- For each contact point in the collision, the velocity after the collision is required to be at least $-\epsilon$ times the relative velocity at the contact point prior to the collision. Where ϵ is the coefficient of restitution. This empirical value is used to describe the nature of the collision. When $\epsilon = 1$ no energy is lost and the collision is perfectly elastic. If $\epsilon = 0$ the collision is perfectly inelastic and the relative velocity of at least one contact point, after the collision, will be zero. This constraint can be written compactly as:

  $$v^+ \geq -\epsilon v^-, \tag{12}$$

  where v^-,v^+ is the relative velocity vector of the contacts before and after the collision.

- The impulse forces at the contact can only push but not pull on the obstacles. If we define a contact space vector p where the i^{th} entry of p represents the impulse force at contact c_i in the n_i direction. This constraint can be compactly written as

  $$p \geq \vec{0}. \tag{13}$$

- The first two constraints place a lower bound on the contact space impulse. The final constraint limits the magnitude of the impulse forces by requiring that if v_i^+ actually exceeds $-\epsilon v_i^-$ at any contact point then the impulse p_i at that contact point must be zero. In contact space this constraint can be written as

  $$p^T(v^+ + \epsilon v^-) = 0. \tag{14}$$

  We will often refer to contact points where $p_i > 0$ as being *active* while points where p_i is zero are referred to as *inactive*.

[1]For brevity we will often write $v \geq \vec{0}$ when $v_i \geq 0 \quad \forall v_i \in v$.

The contact space impulse p causes an instantaneous change in the velocity of the colliding objects. The amount of change depends on the effective mass of the objects seen at the contact points. This relation is given by

$$p \stackrel{\text{def}}{=} \Lambda \Delta v, \tag{15}$$

where $\Delta v = v^+ - v^-$ is the change in contact space velocity caused by the collision. Using Δv condition 12 can be rewritten as

$$\Delta v \geq -(1+\epsilon)v^-. \tag{16}$$

Combining equations 15 and 16 and noting that $v^- = J_c\dot{q}^-$ we can now write constraints 12,13 and 14 as

$$\Lambda^{-1}p \geq -(1+\epsilon)J_c\dot{q}^-, \tag{17}$$

$$p \geq \vec{0}, \tag{18}$$

$$p^T\left(\Lambda^{-1}p - (1+\epsilon)J_c\dot{q}^-\right) = 0. \tag{19}$$

These equations form the constraints to a linear complementarity problem (LCP) that has a form identical to that given by Baraff for simple rigid body simulation. These constraints can be used, as in [3] to solve for the unknown impulse vector p.

Once the values for the impulse forces are known they can be applied to the system. It is easy to show (Appendix A) that the change in configuration space velocities $\Delta\dot{q}$ created by a contact space impulse p is given by

$$\Delta\dot{q} = A^{-1}J_c^T p. \tag{20}$$

The overall system state can now be updated with the new configuration space velocities

$$\dot{q}^+ = \dot{q}^- + \Delta\dot{q} = \dot{q}^- + A^{-1}J_c^T p \tag{21}$$

and the integration of the equations of motion continued from this updated state.

The largest benefit of describing the constraint equations in contact space is that the interaction between group of articulated systems can be described with the same ease as the interaction of a simple rigid body with the ground. Another added benefit is that the constrained equations of motion can now be more readily examined and understood.

An insight into the collision constraints can be found by looking at the contact space consisting of only the *active* contact points (i.e. points where $p_i > 0$). We can, as we did in section 3, write an expression for the reduced Jacobian $J_\ominus$ for this sub-space. This Jacobian is the same as the original contact space Jacobian J_c above with the rows corresponding to the *inactive* contacts eliminated. Expression for the other contact space terms, such as, $v_\ominus$, $\Lambda_\ominus$, and $p_\ominus$ can also be trivially found. Since this space contains only *active* contacts $v_\ominus^+ = -\epsilon v_\ominus^-$, and equation 17 becomes

$$\Lambda_\ominus^{-1} p_\ominus = -(1+\epsilon) J_\ominus \dot{q}^- . \tag{22}$$

In this space $J_\ominus$ is of full rank, having no redundant rows. This implies that $\Lambda_\ominus^{-1}$ is strictly positive definite and hence invertible. This allows us to obtain a direct equation for $p_\ominus$.

$$p_\ominus = -(1+\epsilon) \Lambda_\ominus J_\ominus \dot{q}^- . \tag{23}$$

We can now insert $p_\ominus$ into equation 21 and obtain a linear expression for $\dot{q}^+$ from $\dot{q}^-$

$$\dot{q}^+ = \left[I - (1+\epsilon) A^{-1} J_\ominus^T \Lambda_\ominus J_\ominus \right] \dot{q}^- . \tag{24}$$

This can be written as

$$\dot{q}^+ = \left[I - (1+\epsilon) \overline{J}_\ominus J_\ominus \right] \dot{q}^- , \tag{25}$$

where $\overline{J}_\ominus = A^{-1} J_\ominus^T \Lambda_\ominus$ is the dynamically consistent generalized inverse of $J_\ominus$. This is the unique generalized inverse of $J_\ominus$ that is consistent with the natural dynamics of the system [16].

Separating the ϵ term we obtain

$$\dot{q}^+ = \overline{J}_\ominus \left(-\epsilon J_\ominus \dot{q}^- \right) + \left[I - \overline{J}_\ominus J_\ominus \right] \dot{q}^- . \tag{26}$$

The matrix $\left[I - \overline{J}_\ominus J_\ominus \right]$ is the basis of the null space of the contact space. Velocity vectors mapped through this space do not effect the contact space velocities. A example of the separation between the contact space and its null space is shown in Figure 3

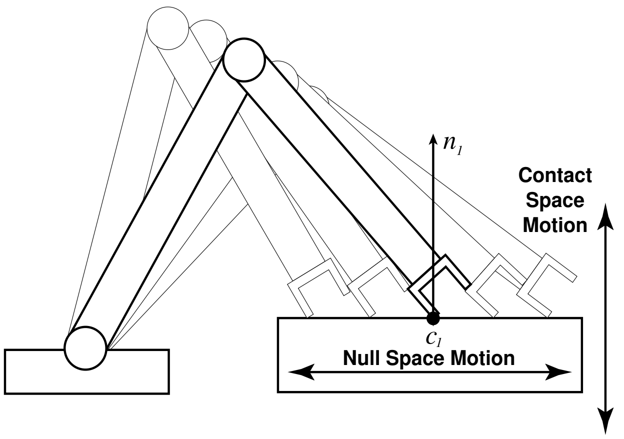

Figure 3: Separation of the local contact motion into contact space and null-space components.

Lastly by noting that $-\epsilon J_\ominus \dot{q}^- = -\epsilon v_\ominus^- = v_\ominus^+$ we see that

$$\dot{q}^+ = \overline{J}_\ominus v_\ominus^+ + \left[I - \overline{J}_\ominus J_\ominus \right] \dot{q}^- . \tag{27}$$

In this expression we can clearly see the effect of the collision constraints on the simulated system. The configuration space velocity after the collision is made up of two components:

- The configuration space velocities that realizes the desired empirical restitution velocity $v_\ominus^+$ with the minimum amount of kinetic energy $K = \frac{1}{2} \dot{q}^T A \dot{q}$ [16].
- Plus the prior configuration space velocities q^- mapped through the contact space null space $\left[I - \overline{J}_\ominus J_\ominus \right]$ eliminating any motion that would affect the velocities in the contact space.

Expression 27 is very similar to the expression for the cartesian space control of redundant mechanisms [16]. Such a connection with control is natural since the constraint equations are in effect commanding an operational space velocity in the contact space while allowing motions in the redundant degrees of freedom to continue unmodified.

Another useful quantity derivable from the contact space mass matrix is the change in kinetic energy ΔK caused by the collision. This expression is given by

$$\Delta K = \frac{1}{2} (1-\epsilon)^2 \Delta v^T \Lambda \Delta v . \tag{28}$$

As can be easily seen when $\epsilon = 1$ no kinetic energy is lost during the collision (elastic collision). When $\epsilon = 0$ all the energy visible at the contact points is dissipated and at the active contact points the relative velocity $v_\ominus = \vec{0}$. This quantity can be used to determine the magnitude of the sound produced by the collision.

5 Mirtich Impact Dynamics

Baraff's constraint equations do not take into account friction at the contact point during collision. When friction is taken into account, the change in velocity experienced by the bodies becomes more difficult to determine and no general closed form solution is known. Mirtich solves for the unknown impulse forces by integrating (with respect to a special collision parameter γ) the equations of motion, over the instant of time that the objects undergo collision, to obtain the change in velocity of the system. Because friction needs to be taken into account the mass properties tangential as well as normal to the contact surface must be examined. Expanding the contact space definition to include the dynamic properties tangential to the contact surface we define the new contact space Jacobian $J_{c'_i}$ for contact point c_i as

$$J_{c'_i} = R_{n_i} \left([I \quad -\hat{r}_{a_i}] J_a - [I \quad -\hat{r}_{b_i}] J_b \right) \tag{29}$$

where J_a and J_b are the basic Jacobians for the colliding bodies A and B at the contact point i. A rotation matrix R_{n_i} is used to rotate the contact surface normal n with the z contact space axis. As in section 3 we can construct the 3×3 operational (contact) space mass matrix

$$\Lambda_{c'_i}^{-1} = J_{c'_i} A^{-1} J_{c'_i}{}^T . \tag{30}$$

As in equation 15 we can now write the relationship between the change in contact space velocities and contact space impulses as

$$\Delta v_{c'_i} = \Lambda_{c'_i}^{-1} p_{c'_i} \tag{31}$$

This relation is identical in form to that presented by Mirtich in [8] where this relation was written as $\Delta u(\gamma) = Mp(\gamma)$. In this equation it can be easily seen that $\Delta u = \Delta v$ is the relative contact space velocity and $M = \Lambda_{c'_i}^{-1}$ is the inverse operational space mass matrix for the contact point. Equation 31 can be differentiated with respect to the collision parameter $\gamma \stackrel{\text{def}}{=} p_{c'_{i_z}}$ and integrated over this parameter (taking into account the effects of friction) to obtain the change in relative velocity caused by this collision. The details of this approach are discussed in [8]. Once the required impulse forces have been found they can be applied to the system using equation 21.

While Mirtich [8] treats multi-contact collision as a series of single point collisions (propagation model) it is easy to see that by stacking the contact space Jacobians as in equation 9 the impact equations can be expanded to handle multiple simultaneous collisions. Additional book keeping is required since the points of maximum compression and restitution for each contact will, in general, not be the same.

6 Contact Force Resolution

In Mirtich's simulation model resting/sliding contact (where the relative velocity normal to contact surface is zero) is not permitted. If $\Delta v = \vec{0}$ a small but bounded impulse is applied to keep the bodies in free space ballistic motion. As was the case with collisions Baraff gives three constraints that govern contact when $\Delta v = \vec{0}$. These are

- The contact forces do not allow the bodies to inter-penetrate.
- The contact forces can "push" but not "pull."
- The contact forces occur only at contact points; once two bodies have separated at a contact point, there is no force between them at that contact point. It can be shown that is constraint is equivalent to the requirement that the contact forces do no work.

Since $v = \vec{0}$ by our assumption it is sufficient to satisfy the first constraint by ensuring that the relative acceleration is greater or equal to zero. The contact space acceleration vector can be found by differentiating equation 10

$$a = \dot{v} = \frac{d}{dt}(J_c\dot{q}) = J_c\ddot{q} + \dot{J}_c\dot{q}. \tag{32}$$

Using equation 2 we can expand $\ddot{q}$ to obtain

$$a = J_c A^{-1}\left[\Gamma - b - g\right] + \dot{J}_c\dot{q} \tag{33}$$

The generalized torque vector Γ is a composite of both internal torques Γ_joint, such as actuator torques, and the external torques Γ_{ext} applied to the system by the environment. The contact space forces and configuration space torque vector is related by [5]

$$\Gamma_{ext} = J_c^T f. \tag{34}$$

Substituting $J_c^T f + \Gamma_{joint}$ for Γ in equation 33 we can write the contact space acceleration as

$$a = J_c A^{-1} J_c^T f + J_c A^{-1}\left[\Gamma_{joint} - b - g\right] + \dot{J}_c\dot{q} \tag{35}$$

The left hand terms which we can denote by $a_{free} = J_c A^{-1}\left[\Gamma_{joint} - b - g\right] + \dot{J}_c\dot{q}$ represents the contact space acceleration if no contact were to occur. Finally using equation 11 we see that the contact space acceleration is a linear function of the contact forces f.

$$a = \Lambda^{-1} f + a_{free} \tag{36}$$

Given a contact space acceleration a that is a linear function of the contact spaces forces f it is possible to write the contact constraint equations in the same form as Baraff [1]:

$$\Lambda^{-1} f \geq -a_{free}, \tag{37}$$

$$f \geq 0, \tag{38}$$

$$f^T(\Lambda^{-1} f + a_{free}) = 0. \tag{39}$$

This LCP constraint system can be used to solve for the unknown contact space forces f as in [3]. Once these forces are known the configuration space acceleration $\ddot{q}$ is given by

$$\ddot{q} = A^{-1}(q)[\Gamma_joint + J_c^T f - b(q, \dot{q}) - g(q)] \tag{40}$$

and we can continue the simulation.

As with the impulse force resolution situation it is interesting to examine the contact sub-space consisting only of the *active* contact points, where $f_i > 0$. In this space:

$$a_\ominus = \Lambda_\ominus^{-1} f_\ominus + a_{\ominus(free)} = \vec{0}. \tag{41}$$

$\Lambda_\ominus$ being positive definite allows us to express $f_\ominus$ as

$$f_\ominus = -\Lambda_\ominus a_{\ominus(free)}. \tag{42}$$

Substituting from equation 11 we can obtain an expression for the change in configuration space acceleration caused by the contact

$$\Delta\ddot{q} = A^{-1}J_\ominus^T f_\ominus = -A^{-1}J_\ominus^T \Lambda_\ominus a_{\ominus(free)} = \overline{J}_\ominus(-a_{\ominus(free)}). \quad (43)$$

This states, in effect, that the negative contact space accelerations mapped through the dynamically consistent generalized inverse are added to unconstrained configuration space accelerations to eliminate the accelerations in the contact space.

7 Contact Space Extensions

The contact space formulation allows other parameterizations of the contact space to be easily incorporated into a simulation system. For example by specifying a configuration space constraint, joint limits can be trivially added to a simulation. Given a joint specified by the configuration space parameter q_i which a joint range a range $q_l \leq q_i \leq q_u$ we can define

$$J_c = [0_1 0_2 \ldots 1_i \ldots 0_m] \quad \text{if } q_i = q_u \quad (44)$$

and

$$J_c = [0_1 0_2 \ldots -1_i \ldots 0_m] \quad \text{if } q_i = q_l \quad (45)$$

The collision and contact constraint equations follow immediately with no additional further changes to the contact space description.

Another case where a different contact space parameterization is beneficial occurs when the polygonal contact region assumption is not valid. Such a situation occurs, for example, when the bottom of a cylinder comes in contact with a plane. In such a situation an infinite number of extremal contact points exist. The contact can alternatively be specified as one prismatic constraint along the surface normal and two rotational constraints. The constraint system can now be solved for unknown constraint forces and torques. If the rotational torques produce zero rotational contact space acceleration then the cylinder is resting on the surface. Otherwise the cylinder is in the process of tipping. If this occurs a single contact point in the direction of the acceleration can be introduced to identify the point of rotation on the contact boundary.

8 Experimental Results

The constraint equations developed above have been used to develop a dynamic modeling system to simulate interactive robotic environments. Using this simulator [17], a wide-range of robotic environments can be represented and simulated. Figure 1 shows as example of a simulated interactive environment, an operational space controller is used to control a robot to topple a set of dominos.

In Figure 4 a simple construction environment is depicted. In this simulation the back-hoe is commanded to move down until it makes contact with the ground

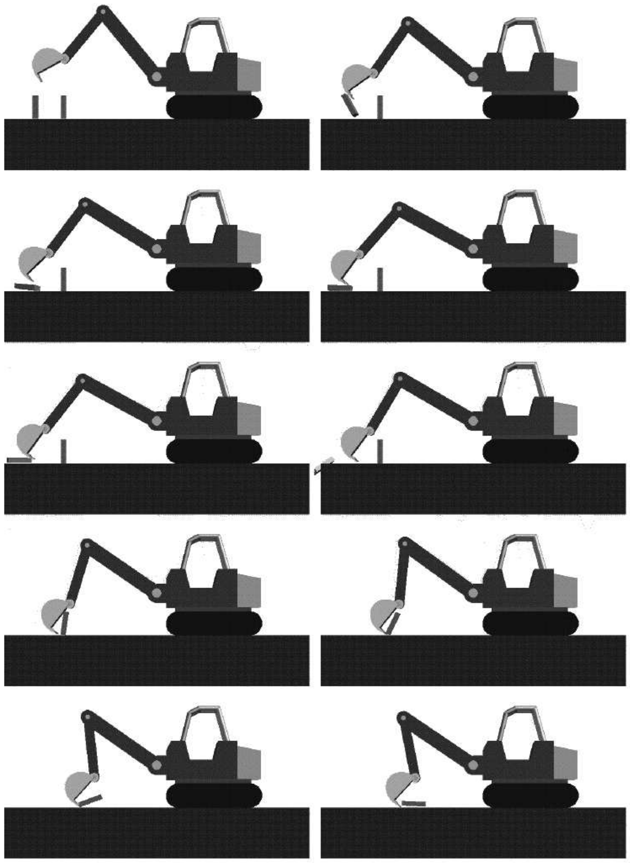

Figure 4: A Sequence of Interactive Dynamic Simulation

and then follow the surface of the floor. Two obstacles unknown to the back-hoe controller have been placed in the path of the shovel. In the sequence shown, the first obstacle is knocked down and the back-hoe proceeds to move along the surface of the block until it reaches the edge of the obstacle where it drops to the floor while continuing its force controlled horizontal motion along the ground. During this movement it hits and knocks down the second obstacle pushing it away as it progresses. Other environments that have been successfully simulated include, a autonomous underwater vehicle docking operation, a tower crane simulation and the simulation of several multi-arm robot work-cells.

9 Conclusion

We have presented a framework for modeling the interaction of complex articulated systems. With this methodology only the external impulse and contact forces on an articulated system need to be computed. Generalize coordinates allow us to ignore the internal forces of a system during the simulation. The contact space formulation allows the constraint equations to be easily specified, and additional constraints added without altering the overall structure of the constraint equations. This framework also hilights the connection between the models used for simulation and control and, it is hoped, will eventually allow even more complex and interesting interactions to be modeled.

Appendix

A

$$p = \Lambda \Delta v \tag{46}$$

applying equation 10

$$p = \Lambda J_c \Delta \dot{q} \tag{47}$$

expanding Λ (equation 11) we obtain

$$p = (J_c A^{-1} J_c^T)^{-1} J_c \Delta \dot{q} = J_c^{-T} A J_c^{-1} J_c \Delta \dot{q} = J_c^{-T} A \Delta \dot{q} \tag{48}$$

multiplying both sides by $A^{-1} J_c^T$ we obtain the expression for $\Delta \dot{q}$

$$\Delta \dot{q} = A^{-1} J_c^T p \tag{49}$$

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