The Effect of Sensor/Actuator Asymmetries in Haptic Interfaces

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Abstract

Haptic interfaces enable us to interact with virtual objects by sensing our actions and communicating them to a virtual environment. A haptic interface with force feedback capability will provide sensory information back to the user thus communicating the consequences of his/her actions. The quality and complexity of these interactions is dependent on how the interface is designed. When designing a haptic interface, one must choose how many sensors and how many actuators will be used. In particular, we are now seeing interfaces which have more sensors than actuators. This "asymmetry" in sensor/actuator utilization provides for a higher dimensionality of action than sensory feedback. It is a tempting avenue for devices design due to the low cost of introducing more sensors. Yet, while this can enable more rich exploratory interactions, the lack for equal dimensionality in force feedback can lead to interactions which are energetically non-conservative. in this paper we provide a preliminary view of the properties of such "asymmetric" sensor/actuator designs. We address the design and rendering tradeoffs of these systems and introduce a framework for device analysis.

1. Introduction

In the last decade the haptic community has grown from being a small sub-set of the robotics community to an established community on its own. Haptic devices are now produced by several companies and have quietly entered several mass markets (automotive, gaming, CAD and design, desktop applications, medical applications, training, etc.). Many devices can now be found, ranging in price, mechanical complexity and usability, from simple low-cost one degree of freedom (DOF) handles used on cars [17] to very complex multi degree of freedom hand exoskeletons [3, 17].

Current state of the art haptic devices can be roughly divided in two main groups. Simpler devices are typically cheaper and more transparent. The level of usability, i.e. the interaction metaphor that can be rendered using such devices, is often limited. More complex devices, on the other hand, are typically harder to build and, as a result, can to be less transparent and more expensive. Their level of usability is however considerably higher. The vast majority of commercially available devices belong to the former class.

A striking example of the limitations in usability of current state of the art haptic devices is represented by the simulation of grasping. Grasping is one of the basic haptic modes [10] and is a key in most types of interaction between humans and the world surrounding them. Very few haptic devices however allow the simulation of such basic ability. While various applications have been created using combinations of 3 DOF desktop devices [9, 1, 8, 4] to the authors' knowledge only one desktop device currently allows the simulation of grasping [15].

The lack of devices allowing such basic capabilities is mainly due to the difficulties in designing transparent devices with high number of degrees of freedom. Sensors usually don't pose a problem for transparency, being often very small and light. Actuators on the other hand strongly limit the transparency of a haptic device due to their low power to weight ratio. Quoting V. Hayward [6] " A haptic device must be designed to *read and write* to and from the human hand (or foot, or other). As it turns out, the *read* part is relatively easy to achieve and a great many types of devices already exist (knobs, keys, joysticks, pointing devices, etc.) although many issues are still unresolved. The *write part* is comparatively much more difficult to achieve".

In this work we will consider the case of under-actuated haptic devices. More specifically we will try to answer the following question: how should we design a haptic device if there is an upper limit on the number of actuators that can be employed?

The reminder of this paper is organized as follows. Section 2 presents some definitions that will be used throughout the paper. Section 3 will analyze the performance of asymmetric (or under-actuated) devices. Section 4 will present two simple explanatory examples. Section 5 will present a device designed to take advantage of alternative principles discussed in the paper. Finally, section 6 will draw some conclusions on the presented work and propose future work opportunities. Note that some important background material is presented in the appendix (section 8).

2 Some definitions

Our discussion will be made in the context of impedance devices [19] (or isotonic devices [6]), i.e. mechanical devices, typically featuring low inertia and high backdrivability, configured to render a commanded force while providing a measurement of their position and/or velocity. An example of such class of devices is the PHANTOM[18]. A similar analysis can be made for admittance devices.

The logical structure of an impedance device is presented in Fig. 1 where x_r is the position on the active ends of the haptic device (HD), x_v is the position of the avatars controlled by the user inside the VE, F_v are the forces returned by the VE to such avatars and F_r are the forces that are actually displayed at the HD active ends.

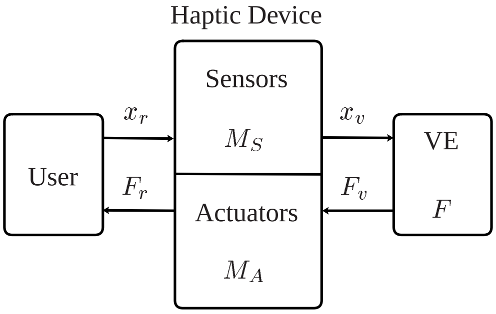

Figure 1. Possible structure for a haptic-based application with a static VE

The logical function of sensors and actuators becomes clear in Fig. 1. Sensors can be seen as input signals from the user to the virtual environment (VE). Actuators on the other hand can be seen as output signals from the VE to the user.

Let us define an avatar as a virtual representation of the user through which physical interaction with the VE occurs. The user controls the avatar position inside the VE. Each sensor allows the user to move its avatar inside the VE along a single degree of freedom twist[1]. When contacts between the user's avatar and the VE arise, action and reaction forces occur. Each HD motor then maps a single degree of freedom wrench from avatar to user. Let us suppose that n is the total number of variables needed to describe an avatar position as well as the contact wrench applied to the avatar by the VE. Possible examples of avatars follow. A single point is an avatar with $n = 3$ since it can only move linearly and can only exchange linear forces with a VE. Alternatively a soft-finger [14] is an avatar with $n = 4$ since it can exchange linear forces and a frictional torque with the VE and thus four variables are needed to describe its position. A set of five single points can be also seen as an avatar (for instance representing the fingertips of a hand). In such case $n = 15$. A rigid body is an avatar with $n = 6$ since six variables are needed to describe its position in space and the net wrench applied to it.

We will borrow the term controllability and observability from control theory; our use is only loosely related to the more formal definition found in that field. In this framework we define *controllability* of a HD as

$$k = \frac{s}{n} \tag{1}$$

where s represents the number of sensors for the device. Similarly we define *observability* of a HD as

$$o = \frac{r}{n} \tag{2}$$

where r is the number of actuators for the haptic device.

Controllability as we use it represents the capacity of the user to control its avatar movements in the VE and thus to exert independent wrenches on the virtual objects. If $k = 0$ (no sensors) the user has no control on its avatars, i.e. cannot control any force on the VE. If $k = 1$ the user has full control on its avatars. *Observability* on the other hand represents the capacity of the VE to exert independent wrenches on the user. If $o = 0$ (no motors) no contact wrenches can be perceived by the user. If $o = 1$ the user can perfectly perceive any wrench system due to contacts between its avatars and the VE.

Referring again to Fig. 1, the sensors of a HD can be described using matrix M_S, which maps the positions of the HD active ends to the positions of the virtual avatars controlled by the user inside the VE. For simplicity, and without loss of generality, M_S can be thought of as an $n \times n$ diagonal matrix. Note that

$$k = \frac{\mathrm{rank}(M_S)}{n} \tag{3}$$

Dual considerations can be made for matrix M_A and therefore

$$o = \frac{\mathrm{rank}(M_A)}{n} \tag{4}$$

In the following with the term *symmetric* device we will refer to HD that have equal number of sensors and motors

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[1] We use the terms wrench and twist to signify generalized forces and motions, respectively, as defined in [14]

an as a consequence $k = o$. Devices for which $k \neq o$ or $s \neq r$ will be referred to as *asymmetric*. While an asymmetric device can be either under-actuated or under-sensed we will only consider the former case.

An example follows. Let us consider three widely available haptic devices: a 3DOF Delta device [16, 13], a Desktop PHANTOM and a 6DOF PHANTOM device [18]. The first of the above HD has three motors and three sensors and is normally used to simulate a single-point interaction. Thus both M_S and M_A have full rank, the device is perfectly controllable and observable and therefore is symmetric. The second of the above devices has six sensors and three motors. If it is used to control a single point of contact then $n = 3$, M_S and M_A are 3×3 full rank matrices and the device is symmetric. If however the device is used to control a rigid tool then $n = 6$, M_S and M_A are 6×6 matrices and only the former has full rank. In such case the device is not fully observable a and is asymmetric. The third of the above devices can be used to control a rigid tool. In such case $n = \mathrm{rank}(M_A) = \mathrm{rank}(M_S) = 6$, i.e. the device is symmetric.

3 Haptic devices with limited number of motors

Let us now consider the case of designing a haptic device given a fixed set of motors. In the following we will assume that sensors are “free”, i.e. can be used without compromising the overall level of transparency of the device.

Let us suppose that for a particular haptic application n DOF are needed to control the user’s avatars. If the number of motors that can be used by the designer is equal to n, a fully controllable and observable HD can be designed. However this will typically not be the case. A normal PHANTOM device, for instance, does not have enough motors to fully simulate the forces that would be created in the case of two single-point contacts grasping a virtual object.

What happens in the case of $r < n$? Two possible approaches can be used. If we decide to use equal number of sensors and actuators ($s = r < n$) then the device is symmetric. However the user cannot fully control its avatars and as a consequence the interaction metaphor with the VE is strongly limited. Using more sensors than actuator ($s = n > r$), on the other hand, leads to an asymmetry in the device. In this case the device is fully controllable, i.e. the user has complete control on its avatars and can apply any wrench system (allowed by such avatars) to the VE. However the device does not allow full observability, i.e. not all wrenches applied to the VE by the avatars can be displayed back to the user. [2]

What are the advantages and disadvantages of the above approaches? In the following we will show that extra controllability enhances the observability of a device, i.e. the device may appear more capable of displaying interaction forces than it really is. This however comes for a price. The level of realism that can be obtained is in fact, in certain situations, more limited. More specifically conservative force systems, like the ones modeled using springs, may become not conservative.

In order to better explain such phenomena let us consider a static frictionless VE. Let us suppose, for simplicity, that the user controls a set of p contact points[3], i.e. $n = 3 \times p$. The contact of such points with the VE can be modeled by a spring-based proxy algorithm [12, 20], i.e. by a positional system of forces

$$F : R^n \rightarrow R^n \tag{5}$$

which is conservative (see the Appendix for more on positional and conservative forces). As a consequence of this the matrix representing the linearization of F, given by

$$F_L = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \dots & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \tag{6}$$

where

$$F(x) = \begin{bmatrix} F_1 \\ \vdots \\ F_n \end{bmatrix} \quad \text{and} \quad n = 3 \times p \tag{7}$$

is symmetric.

In order to fully simulate the interaction between such points and the VE we need a device with $s = r = 3 \times p$. If $p > 1$ this can become very complex. Such complexity can be limited by limiting the number of motors in the HD ($r < n$). In such case the linearization of the positional force system due to contacts between avatars and VE that can be displayed by the HD, expressed by matrix

$$M_A \, F_L \, M_S \tag{8}$$

where has the following characteristics:

- if $\mathrm{rank}(M_A) = \mathrm{rank}(M_S) = n$ then the HD is capable to perfectly display the interaction between avatars and VE. Matrix $M_A \, F_L \, M_S$ is symmetric, i.e. the forces perceived by the user through the HD are still conservative.

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[2] In a sense this can be seen as an asymmetry between action and reaction between user and VE, i.e. a situation where Newton’s third law is not satisfied.

[3] Each contact point is described by three variables and is capable of point contact [14], i.e. can exchange a linear force with the VE expressed by three variables.

- if $\mathrm{rank}(M_A) = \mathrm{rank}(M_S) < n$ then matrix $M_A\,F_L\,M_S$ is still symmetric but does not have full rank anymore. The system of contact forces displayed by the HD is conservative. However controllability and observability are limited and therefore the user cannot fully “exploit” its avatars. The metaphor presented to the user is simpler.
- if $\mathrm{rank}(M_A) = r < \mathrm{rank}(M_S) = n$ then the HD is asymmetric. The controllability of the device is still perfect while the observability is limited. It is important to note, however, that the controllability enhances the observability of the device. In fact matrix $M_A\,F_L\,M_S$ has now the following form

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \cdots & \cdots & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial F_r}{\partial x_1} & \cdots & \cdots & \cdots & \cdots & \frac{\partial F_r}{\partial x_n} \\ O & \cdots & \cdots & \cdots & \cdots & O \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ O & \cdots & \cdots & \cdots & \cdots & O \end{bmatrix} \qquad (9)$$

  where we have supposed, without loss of generality, that the first r elements on the diagonal of M_A are non-zero. Note that the existence of null elements on the diagonal of matrix (9) implies that directions exist along which no stiffness can be directly perceived, i.e. when moving along such directions the user will not be able to feel a reaction force along the same direction. However due to the non-null elements off the diagonal in the first r rows of the matrix the user is always able to perceive the projection of any contact force along the set of directions covered by the device actuators. The drawback, however, is that matrix (9) is not symmetric anymore and thus system of contact forces displayed by the HD is no longer conservative. Thus the VE tends to feel either too active or too passive, depending on the trajectory described by the avatars.

Summarizing, the matrix that describes the positional force system that can be displayed by an asymmetric devices is also asymmetric. Similarly symmetric devices feature symmetric matrices.

4 A simple example

Two explanatory examples are presented in the following.

4.1 Single point on tilted wall

Consider the 2DOF VE depicted in Fig. 2. The VE implements a simple virtual wall without friction. The user can control a point inside the VE. Such point interacts with the wall using a simple proxy-based contact model. Let us suppose that the plane is tilted around an axis perpendicular to the $x - y$ plane. Let α be the angle between the virtual plane and the x axis. The avatar has 2 DOFs, i.e. $n = 2$.

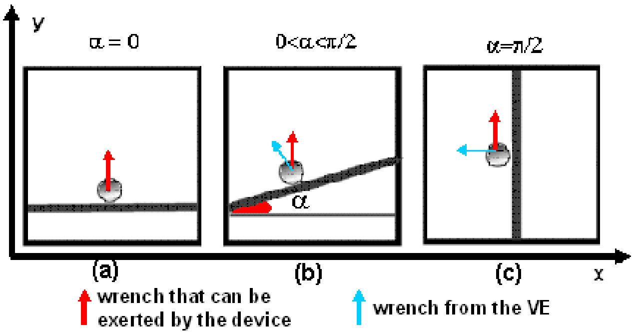

Figure 2. A 2DOF VE composed by a virtual wall which can be described by equation $y = tg(\alpha)\,x$.

In order to perfectly simulate this VE we need a HD with $r = s = 2$. In such case $k = o = 1$ and the accomplished level of realism is maximized.

We now consider what can be simulated with devices with a single motor. If the HD has 1 motor and 1 sensor, both acting along the same direction, the overall metaphor is simplified to the case of a 1DOF virtual wall, i.e. the tilt of the wall cannot be perceived. However the interaction with such wall is fully realistic.

Let us now suppose that the HD has 1 motor (capable of exerting forces along the y) and 2 sensors ($o = 1 > k = 0.5$). The user can still move in a 2DOF VE and exert forces on the wall along both x and y. The user can only feel forces along the y axis. However by being able to move in 2D the user can feel the projection of the contact force along the y axis, i.e. the user is able to perceive the inclination plane. Such projected component grows further apart from the correct interaction force as α grows. More specifically when $\alpha = 0$ rad (Fig. 2 (a)) the haptic device is capable of exerting the correct force on the user since the only wrench the user can exert on the VE is along the y axis. When α grows (Fig. 2 (b)), however, the device cannot prevent the user from penetrating the plane when moving along the x axis. Because of the nature of impedance displays, and because sensor resolution and servo rates are limited, the user will penetrate to a position where it normally would not get and is then brusquely pushed out of the object along the y axis. The motion of the user hand still describes the contour of a tilted plane: the user still has the impression of touching a tilted plane. The contact can, however, feel strangely active or passive. The level of realism decreases while α increases. At the limit case of $\alpha = \frac{\pi}{2}$ (Fig. 2 (c)), when the wrench system that the VE exerts on the user is perfectly orthogonal to the wrench system that the HD can display, the level of realism is completely lost and no force is exerted on the user.

In a more formal way the components along x and y of the proxy-based contact force $F(x, y)$ relative to the VE described in Fig. 2 are described by

$$\begin{cases} F_x(x, y) = k\,(\frac{\sin(2\alpha)}{2}y - \sin(\alpha)^2 x) \\ F_y(x, y) = k(\frac{\sin(2\alpha)}{2}x - \cos(\alpha)^2 y) \end{cases} \tag{10}$$

when $y < \tan(\alpha)\,x$ while they are both null otherwise. The matrix representing the linearization of F is therefore

$$F = \begin{bmatrix} -k\sin^2(\alpha) & k\sin(\alpha)\,\cos(\alpha) \\ k\sin(\alpha)\,\cos(\alpha) & -k\cos^2(\alpha) \end{bmatrix} \tag{11}$$

In case two sensors and one actuator (capable of exerting forces along the y axis) are used, matrix $M_A\,F_L\,M_S$ is given by

$$\begin{bmatrix} O & O \\ k\sin(\alpha)\,\cos(\alpha) & -k\cos^2(\alpha) \end{bmatrix} \tag{12}$$

and the force systems due to contacts with the VE is not conservative anymore. Note that in the case of $\alpha = 0$ the matrix results symmetric and in fact the interaction with the VE can be perfectly replicated by the HD, since the only forces that can be exchanged with the VE are parallel to the ones that can be exerted by the HD. When α grows the matrix tends to become more and more asymmetric, i.e. element $(2, 1)$ grows, and the system of forces less conservative. Finally when $\alpha = \pi/2$ all the elements in matrix $M_A\,F_L\,M_S$ are equal to zero and no interaction force can be exchanged with the VE. The amount of work over a closed trajectory of the form described in Fig. 3, in the latter case of asymmetric HD, is given by

$$\Delta\,L^2\,\sin^2(\alpha)/2 \tag{13}$$

i.e. it is proportional to how far the user has penetrated along the $x-$axis before the actuators have been able to react along the $y-$axis. The surface feels active since the proxy stores a certain potential energy (along the $y-$axis) without doing any work along the $x-$axis. Such amount of work is dependant on how fast the user is moving along x, on how fast the HD reacts (i.e. depends on servo-rate and sensor resolution) and on α.

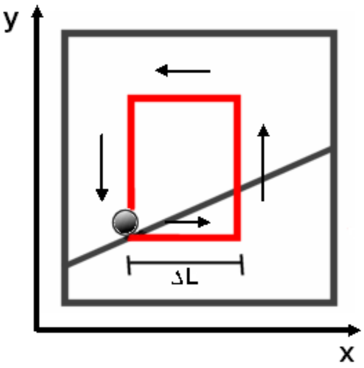

Figure 3. Contact force work over a closed trajectory

The simple scenario described in Fig. 2 has been anecdotally investigated using a PHANTOM as a two DOF HD on a set of six users. At first the users were allowed to move freely in a two DOF VE using the PHANTOM as a symmetric device. The plane could be tilted of a generic angle $\alpha \in [-\pi/2, \pi/2]$. In a second phase of the experiment, the PHANTOM was used an asymmetric device by disabling force feedback along the x or y axis. The users, who were not informed of this change, were asked to comment on the overall level of realism of the simulation. In both situations visual feedback was also available.

Reactions of the users, which are summarized in table 4.1, match what has been presented above. None of the users realized that the PHANTOM was being used a 1DOF device for most of the experiment, even if this became clear when $\alpha = \pm\pi/2$. All the users did however notice unrealistic effects for values of α larger than $\pi/6$.

α	level of realism
0	perfect
~ 0	almost perfect
$\simeq \pi/6$	slightly unrealistic
$> \pi/6$	markedly unrealistic
$\simeq \pi/2$	completely unrealistic

Table 1. Level of realism that can be accomplished using an asymmetric device

4.2 Two points on static rod

Let us now consider the case of two points touching the two opposite faces of a static wall, as depicted in Fig. 4. Each point can contact only one side of the wall. A spring-based proxy model is considered. This simple scenario roughly represents a user pinching two sides of a wall using index finger and thumb. For simplicity let us suppose that the two points can only move in a 2D plane described by a $x - y$ reference frame. Furthermore let us suppose that the

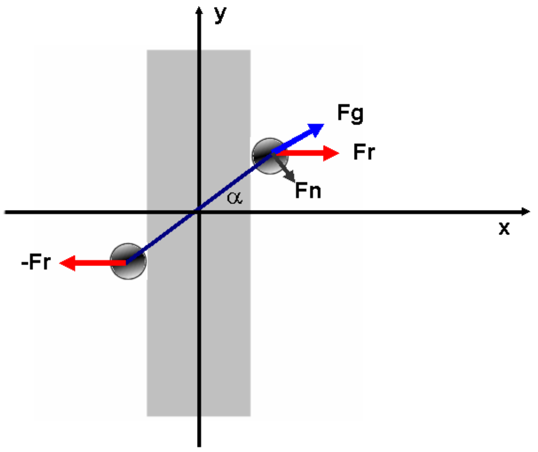

Figure 4. Two points touching two opposite faces of a virtual wall. The user can move the two points by changing angle α and by changing the distance between the two points L.

center of the line connecting the two points controlled by the user is fixed in the origin of the $x - y$ reference frame. Given the above constraint the position of the two points can be described using two variables ($n = 2$): L represents the distance between the two points; α represents the angle between the line connecting the points and the $x-$axis. As a result of contacts with the wall the user will experience a force along the line connecting the two points F_g and a torque τ. If the stiffness coefficient for the wall is k and the right face of the wall is expressed as line $x = x_R$, the positional force system describing F_g and τ with respect to L and α is given by

$$\begin{cases} F_g = k\,(x_R - \frac{L}{2}\cos(\alpha))\cos(\alpha) \\ \tau = -L\,k\,(x_R - \frac{L}{2}\cos(\alpha))\sin(\alpha) \end{cases} \qquad (14)$$

when

$$\frac{L}{2}\cos(\alpha) < x_R \qquad (15)$$

while they are both zero otherwise. This force system is conservative. This can be seen when looking at the matrix (6) obtained by linearizing relation (14)

$$\begin{bmatrix} -\frac{k}{2}\cos^2(\alpha) & k\,(L\frac{\sin(2\alpha)}{2} - x_R\sin(\alpha)) \\ k\,(L\frac{\sin(2\alpha)}{2} - x_R\sin(\alpha)) & k\,L(\frac{L\sin(2\alpha)}{2} - x_R\cos(\alpha)) \end{bmatrix} \qquad (16)$$

which is symmetric.

What haptic device is best suited for interacting with this VE? If no upper limit is set on the number of motors that can be used then a 2DOF HD capable of exerting a linear force along the line connecting the points and a torque around the center of such line can be used to faithfully represent kinematic and energetic interaction with such scene.

If only one motor is allowed to be used the designer has the option to use one or two sensors. In the first case the overall metaphor is greatly simplified and the user does not have the impression to be touching two sides of a thick wall. In the second case the user is able to perceive the shape of the wall through active exploration. However the system of contact forces that can be displayed by this type of device is no longer conservative. If for instance the HD can only exert a F_g force, i.e.

$$M_A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (17)$$

then matrix $M_A\,F_L\,M_S$ becomes

$$\begin{bmatrix} -\frac{k}{2}\cos^2(\alpha) & k\,(\frac{L\sin(2\alpha)}{2} - x_R\sin(\alpha)) \\ 0 & 0 \end{bmatrix} \qquad (18)$$

which expresses the linear approximation of relation 14 as displayed by the proposed HD. Element $(M_A\,F_L\,M_S)_{(1,2)}$ of such matrix allows the user to perceive the effect of a rotation along the direction of F_g. However it is due to this element that the device feels non-conservative.

5 A possible application

In the recent past our research group has been focused on the design of desktop haptic devices that allow dual-handed grasping of virtual objects. More specifically the device allows users to manipulate objects using *palmar pinch* (also referred to in many different ways such as *precision grip*, *writing grip* [2] or *pinch grasp* [7]) of both left and right hand. [4]

The simplest avatar representing a palmar pinch is based on two single-point contacts. Such avatar can be fully described using six variables and therefore a device that perfectly renders such scenario needs six actuators and sensors. This however can strongly limit the level of transparency for the device and increase mechanical complexity and cost. Adding one motor to current state of the art 3DOF devices is the simplest incremental step towards allowing users to grasp virtual objects. While using only four motors will not lead to devices that can accomplish a perfect level of realism, this solution has several advantages. A 4DOF device will typically be simpler, cheaper and more transparent than a 6DOF one. Its performance will in general be

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[4] Palmar pinch can be defined as the hand coupling where a force is exerted between the pad of the index finger and the pad of the thumb, through the centers of the opposing pads.

higher. Moreover such device can be used in conjunction with pre-existing hardware such as PHANTOM or Delta devices. Given an upper bound of four motors, various design choices are allowed as previously described. We have built and tested both of these solutions.

Figure 5. A four 4DOF haptic device that can be used for virtual grasping

In Fig. 5 a 4DOF device comprising four motors and four sensors is depicted. The device is based on a PHANTOM 1.5 and a force reflecting gripper [11] rigidly connected to each other. The user is thus able to interact with a VE using two single points contacts. Such points however cannot be oriented in the VE, i.e. the line connecting the two points is always parallel to itself. The device is symmetric but both $k = o < 1$.

In Fig. 6 a 6DOF device comprising four motors and six sensors is depicted. The device is based on a PHANTOM 1.5 and a force reflecting gripper [5] connected by a sensorized wrist. The user is thus able to fully position the two points of interaction inside the VE and thus apply any wrench, allowed by the particular avatars chosen, on virtual objects. Perfect controllability is accomplished. Observability however is limited, i.e. $o < k = 1$. The device is asymmetric and therefore unrealistic effects are displayed to the user. The device is not capable of exerting any torque on the user. However a projection of such torques is obtained along the force-reflecting gripper line of action. This projection is however responsible for a loss of realism due to overly active or passive device responses, as described in section 4.2.

6. Conclusions

In this paper we discuss some of the advantages and disadvantages in using under-actuated haptic devices. Under-actuated devices tend to be more transparent and cheaper.

Figure 6. An asymmetric 6DOF haptic device that allows users to pinch virtual objects using index finger and thumb

Their usability, however, is often more limited. Using more sensors than actuators can partially overcome the usability issue while not negatively impacting cost or transparency. A consequence of this choice, however, is a loss of realism in certain situations. More specifically while it is still possible to correctly perceive the shape of objects inside the virtual environment, the system of contact forces tends to become non-conservative.

While asymmetry can be an attractive solution it is definitely not feasible in situations where high level of realism is a key issue. Future work will focus on creating haptic rendering techniques for asymmetric devices to partially limit the unrealistic effects described above.

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8. Appendix

Let us first introduce some basic definitions. A system of forces

$$F : \Omega \rightarrow R^n \tag{19}$$

defined on a subspace $\Omega \subset R^n$ is said to be *positional* when it only depends on the position inside Ω, i.e. when

$$F = F(x) = \begin{bmatrix} F_1(x) \\ \vdots \\ F_n(x) \end{bmatrix} \quad \forall x \in \Omega \tag{20}$$

A system of positional forces is said to be *conservative* if a function $U : \Omega \rightarrow R$, referred to as potential energy, exists such that

- *1)* U is differentiable on Ω
- *2)* $dU(x) = F(x) \cdot dx \ \ \forall\, x \in \Omega$

Note that condition (2) is equivalent to having

$$F_i(x) = \frac{\partial U}{\partial x_i} \ \ i \in Z_n. \tag{21}$$

Note that a positional force system F is conservative only if

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i} \ \forall\, i, j \in Z_n,\ i \neq j \tag{22}$$

i.e. condition (22) is necessary but not sufficient for F to be conservative. This implies that the linearization of F along x

$$F_L = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \tag{23}$$

is a symmetric matrix when F is conservative.

One of the most important features of conservative force systems is that their work over any closed trajectory in Ω is always zero, i.e.

$$W = \oint F \cdot dx = 0 \tag{24}$$