Efficient Synaptic Pruning with Neuronal Regulation

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Abstract

Neuronal regulation is a mechanism that was recently found to maintain the homeostasis of the neuron's membrane potential. We show that the operation of this mechanism may lead to a bi-modal distribution of synaptic efficacies, pruning the weak synapses and strengthening the rest. Deriving optimal synaptic modification functions, we identify conditions under which neuronal regulation leads to near optimal modification of synaptic efficacies. These results point to the possible important role of neuronal regulation as a biologically plausible mechanism that governs the fundamental process of synaptic pruning observed in mammalian brain development.

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1 Introduction

This paper studies one of the fundamental puzzles in brain development: the massive synaptic pruning observed in mammals during childhood, which removes more than half of the synapses [Rakic *et al.*, 1994]. Addressing this issue, our work stems from three studies presented in the previous CNS meeting: We have previously shown that synaptic overgrowth followed by judicial synaptic pruning subject to synaptic constraints improves memory performance along development [Chechik *et al.*, 1997, Chechik *et al.*, 1998]. This optimal pruning requires that synapses are pruned according to their efficacy, removing the weaker synapses first, and suggests a new computational explanation for the massive synaptic pruning in early childhood.

The important question remains however, does there exist a biologically plausible mechanism that can actually implement our theoretically derived optimal pruning ? The current work shows that *neuronal regulation*(NR), a mechanism which operates to maintain the homeostasis of the membrane potential of the neuron [Horn *et al.*, 1998, Horn *et al.*, 1997] can serve as a mechanism that selectively prunes synapses depending on their efficacy. Interestingly, the existence of NR has been independently proposed and experimentally identified by [Turrigano *et al.*, 1997]. They showed evidence that neurons both up-regulate and down-regulate the efficacy of their incoming synapses in a multiplicative manner, maintaining their membrane potential as required by NR.

Our main results are twofold: First, the biologically plausible NR mechanism efficiently *removes the weakest synapses* in a manner conserving the network's performance. Second, we identify a crucial parameter (*degradation dimensionality*) whose value determines if NR *modifies the rest of the synapses* in a near optimal manner. Since we propose that NR plays a prominent role in synaptic pruning during development, we predict that this condition is fulfilled in the brain during that period. We thus add to the role of NR in maintaining neuronal firing homeostasis proposed previously by [Turrigano *et al.*, 1997, Horn *et al.*, 1998] a new role, that of guiding pruning during development.

This summary is organized as follows: We first study how NR modifies synaptic efficacies. To this end, we derive the function of new (modified) versus original synaptic values generated by the NR process. The efficiency of the resulting NR modification function is compared to optimal modification functions derived analytically.

2 Synaptic modification with Neural Regulation

We study NR in the general framework of associative memory networks. Memories are first stored in a fully connected network, following a Hebbian rule. Then, synaptic values are continuously modified as the consequence of two concomitant processes: *synaptic degradation*, which is an inevitable consequence of ongoing metabolic turnover [Wolff *et al.*, 1995], and neurally regulated *synaptic maintenance*, operating to compensate for the degradation.

At each degradation/compensation step, synaptic degradation stochastically reduces the synaptic strength by

$$W'_{t+1} = W_t - W_t^{\alpha} * \eta_t \tag{1}$$

where η is a normally distributed noise term and α is the *degradation dimensionality* parameter in the range $[0, 1)$. Then, the post synaptic neuron multiplicatively strengthens all its synapses by a common factor, to retain its original post synaptic potential (PSP).

$$W_{t+1} = W'_{t+1} * \frac{\text{baseline PSP}}{\text{current PSP}} \tag{2}$$

The synapses are assumed to have an upper bound reflecting the maximal synaptic efficacy, and a viability lower bound below which a synapse degenerates and vanishes.

Studying degradation/compensation via numerical simulations yields two main results. First, the system reaches a (thermodynamic) steady-state in which the distribution of synaptic values is bi-modal : Many synapses decrease below the lower bound and are then pruned, and the rest drift toward a high synaptic value, as shown in Figure 1.

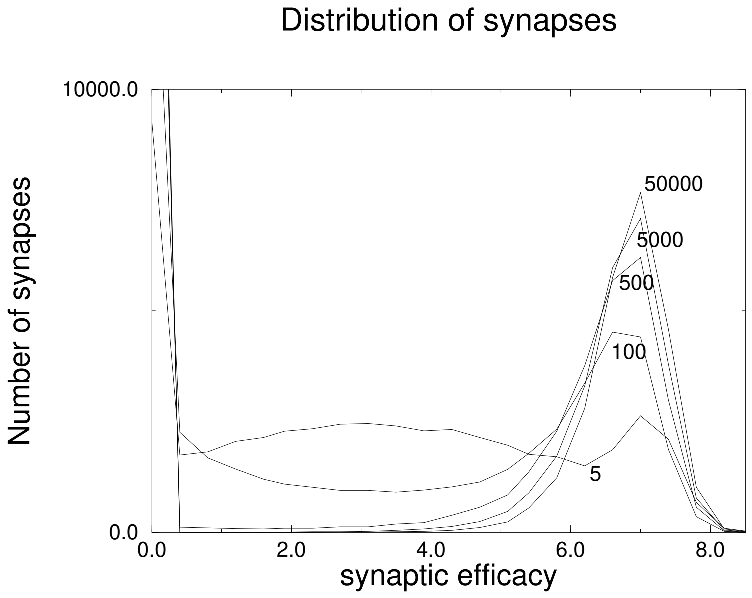
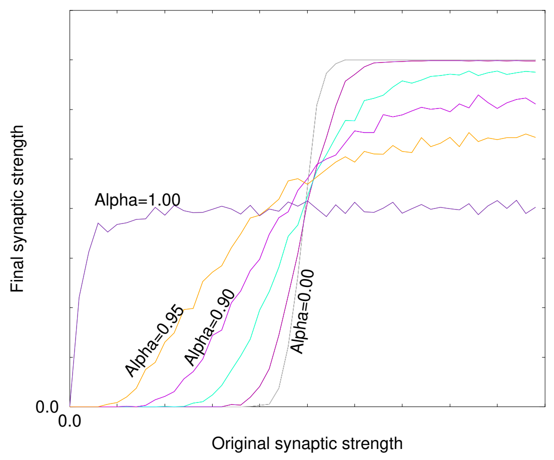


Figure 1: Distribution of synaptic values following a degradation-compensation process, after $5, 100, 500, 5000$ and 50000 steps. Simulation parameters: α=0.9, no of synapses = 50000, low bound = 0.1, upper bound = 7. Qualitatively similar results were obtained for a wide range of simulations parameters.

Second, the resulting steady state modification function strongly depends on the degradation dimensionality α. As can be observed in Figure 2A, the NR synaptic modification functions always prune the weakest synapses with high probability, and apply sigmoid-shaped modification to the remaining synapses, with a slope depending on α.

A. NR modification function for different α values.



B. NR modification function along time.

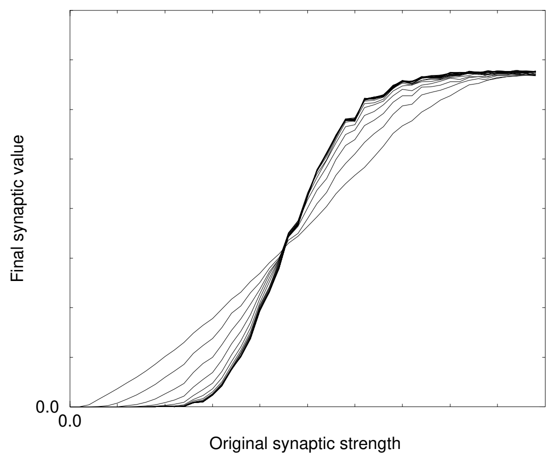


Figure 2: A. Modification function at steady state, for different α values. Results were obtained for a single neuron performing 100000 steps of degradation/compensation on all its 50,000 synapses, for $\alpha = 0.0, 0.5, 0.8, 0.9, 0.95, 1.00$. B. Time evolution of the modification function for $\alpha = 0.8$. Modification functions are plotted each 50 degradation/compensation steps until a steady state is reached within a very fast convergence time.

3 Synaptic Pruning in Excitatory-Inhibitory Networks

We have shown that NR implements a selective pruning mechanism, which removes the weakest synapses and modifies the rest in a sigmoidial manner. But is such a pruning strategy effective? And what is its effect on network performance? To answer these questions we have analytically studied *optimal modification functions* under various synaptic constraints and calculated the resulting network performance.

Our analysis extends the well known method of signal-to-noise analysis of neuronal input fields used in [Chechik *et al.*, 1997], and applies it to a variation of a low-activity biologically-motivated model proposed by [Tsodyks and Feigel'man, 1988]. Memories are stored in a network of inter-connected excitatory neurons via a Hebbian learning rule. Neurons also receive inhibitory input proportional to the activity of the excitatory network. We investigate the effect of a general modification function $g(w)$ over the excitatory synapses, which a) prunes some of the synapses, and b) possibly modifies the rest. As synaptic resources are believed to be costly [Roland, 1993], we look for optimal g functions under constraints of the form $\int g^{\beta}(w)dw$ which limit the synaptic resources. Our results are described below omitting the derivations.

The optimal modification requires pruning of intermediate synapses with linear modification of the non-pruned synapses. However, pruning the weak synapses, and modifying the remaining synapses linearly is near optimal. Figure 3A shows the marked advantage of these two modification functions over random (non-weight dependent) pruning, especially for the high deletion ratios observed in the brain [Rakic *et al.*, 1994].

A. **Capacity under different modification strategies** B. **Comparing modification functions**

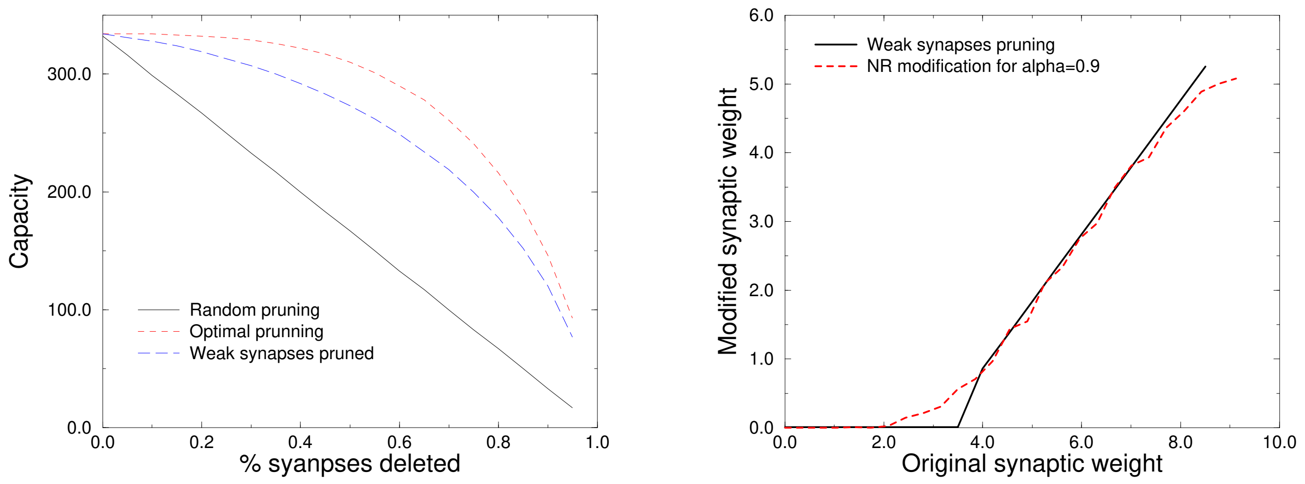


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Figure 3: A. Comparison between performance of different modification strategies as a function of the deletion level (percentage of synapses pruned). Results were obtained analytically for a network of 800 neurons. Capacity is measured as the number of patterns that can be stored in the network and be recalled almost correctly from a degraded pattern. B. NR modification function (with $\alpha = 0.9$) and weak synapses pruning (derived to be optimal with $\beta = 0$), at 50% deletion.

The close similarity between the modification function generated by NR and the weak-synapses pruning strategy is shown in Figure 3B.

4 Summary

Neuronal regulation, a mechanism experimentally found to maintain neuronal firing rates during early development, leads to weight dependent synaptic pruning removing the weak synapses. Interestingly, recent studies in the neuromuscular junction have shown that synapses are either strengthened or pruned during early development, depending on their efficacies [Frank, 1997]. Our analysis suggests that in addition to its developmental role in maintaining membrane potential [Turrigano *et al.*, 1997], neuronal regulation results in near optimal synaptic modification maximizing memory performance.

References

[Chechik *et al.*, 1997] G. Chechik, I. Meilijson, and E. Ruppin. Synaptic pruning during development: A novel account in neural terms. In *Sixth Annual Computational Neuroscience Meeting. (CNS97), Big Ski, Montana.* 1997.

[Chechik *et al.*, 1998] G. Chechik, I. Meilijson, and E. Ruppin. Synaptic pruning during development: A computational account. *Neural Computation. In press.*, 1998.

[Frank, 1997] E. Frank. Synapse elimination: For nerves it's all or nothing. *Science*, 275:324–325, 1997.

[Horn *et al.*, 1997] D. Horn, N. Levy, and E. Ruppin. Dynamic memory maintenaince. In *Sixth Annual Computational Neuroscience Meeting. (CNS97), Big Ski, Montana.* 1997.

[Horn *et al.*, 1998] D. Horn, N. Levy, and E. Ruppin. Synaptic maintenance via neuronal regulation. *Neural Computation*, 10(1):1–18, 1998.

[Rakic *et al.*, 1994] P. Rakic, J.P. Bourgeois, and P.S. Goldman-Rakic. Synaptic development of the cerebral cortex: implications for learning, memory and mental illness. *Progress in Brain Research*, 102:227–243, 1994.

[Roland, 1993] Per E. Roland. *Brain Activation.* Willey-Liss, 1993.

[Tsodyks and Feigel'man, 1988] M.V. Tsodyks and M. Feigel'man. Enhanced storage capacity in neural networks with low activity level. *Europhys. Lett.*, 6:101–105, 1988.

[Turrigano *et al.*, 1997] G.G. Turrigano, K. Leslie, N. Desai, and S.B. Nelson. A biological mehcanism for synaptic stability in developing neocortical circuits. In *Sixth Annual Computational Neuroscience Meeting. (CNS97), Big Ski, Montana. ; Nature, In press.* 1997.

[Wolff *et al.*, 1995] J.R. Wolff, R. Laskawi, W.B. Spatz, and M. Missler. Structural dynamics of synapses and synaptic components. *Behavioural Brain Research*, 66:13–20, 1995.