Climbing Robots in Natural Terrain

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Abstract

This paper presents a general framework for planning the quasi-static motion of climbing robots. The framework is instantiated to compute climbing motions of a three-limbed robot in vertical natural terrain. An example resulting path through a large simulated environment is presented. The planning problem is one of five fundamental challenges to the development of real robotic systems able to climb real natural terrain. Each of the four other areas—hardware design, control, sensing, and grasping—is also discussed.

1 Introduction

The work described in this paper is part of an effort to develop critical technologies that will enable the design and implementation of an autonomous robot able to climb vertical natural terrain. To our knowledge, this capability has not been demonstrated previously for robotic systems. Prior approaches have dealt with artificial terrain, either using special "grasps" (e.g., pegs, magnets) adapted to the terrain's surface or exploiting specific properties or features of the terrain (e.g., ducts and pipes) [1-12].

Developing this capability will further our understanding of how humans perform such complex tasks as climbing and scrambling in rugged terrain. This may prove useful in the future development of sophisticated robotic systems that will either aid or replace humans in the performance of aggressive tasks in difficult terrain. Examples include robotic systems for such military and civilian uses as search-and-rescue, reconnaissance, and planetary exploration.

Many issues need to be addressed before real robots can climb real, vertical, natural terrain. This paper considers five of the most fundamental of these issues: hardware design, control, sensing, planning, and grasping. One of these issues in particular, the motion-planning problem, is described in more detail. A general framework for climbing robots is presented and this framework is instantiated to compute climbing motions of the three-limbed robot shown in Figure 1. Simulation results are shown for the robot in an example vertical environment.

Fig 1. A three-limbed climbing robot moving vertically on natural surfaces.

2 Motivation

The results of research in this area will benefit a number of applications and have implications for several related research areas.

2.1 Applications

This paper is motivated by a need for robotic systems capable of providing remote access to high-risk natural environments.

There are many terrestrial applications for these systems, such as search-and-rescue, cave exploration, human assistance for rock and mountain climbing, and tactical urban missions. Each of these applications requires climbing, descending, or traversing steep slopes and broken terrain, and thus involves considerable human risk.

Several space applications could also benefit from these aggressive robotic systems. For example, sites on Mars with potentially high science value have been identified on cliff faces [13]. Often, it is neither practical nor feasible for flying robots to access these locations. Therefore, to reach these sites, robots must climb, descend, or traverse steep slopes. Future goals for exploration on other planetary bodies may require access to equally rugged terrain.

2.2 Implications

In addition to furthering the development of a climbing robot for vertical natural terrain, the results of research in this area could provide fundamental insight into several related research areas. For example, this study could lead to the development of better strategies for robotic walking or dexterous manipulation. Human climbers often comment on an increase in balance and an expanded range of movement in everyday activity as they become more proficient at the sport. This enhanced mobility is often referred to as "discovering new degrees of freedom," and is related to the idea of discovering useful new modes of mobility for extremely complicated humanoid robots or digital actors.

Also, the development of planning algorithms for climbing robots could lead to a better set of criteria for the design of these types of robots. These algorithms could be applied to candidate designs in simulation to determine the capabilities of the resulting robots, and thus to select a design.

3 Fundamental Issues

There are five fundamental issues involved in climbing steep natural terrain: hardware design, control, sensing, grasping, and planning. A substantial amount of work needs to be done in each of these areas in order to develop a real climbing robot. This section describes the challenges involved in the first four of these areas; the planning problem will be discussed in more detail in Section 4.

3.1 Hardware Design

A good hardware design can increase the performance of the robot, and often can make each of the other fundamental issues easier to deal with. However, past use of hardware solutions in maintaining equilibrium generally resulted in a fundamental limitation on the terrain that could be traversed.

Wheeled robotic systems have been used to ascend and traverse natural slopes of up to 50 degrees, to descend slopes of up to 75 degrees, and to climb over small obstacles in rough terrain. These systems either use some form of active or rocker-bogie suspension as in [12, 14-16], or use rappelling as in [1]. Similar results have been obtained using legged rappelling robots [3, 17] and a snake-like robot [4].

The terrain that these rovers can traverse robustly is impressive, but none of the existing systems has been shown to be capable of climbing natural slopes of 90 degrees or higher. Wheeled rovers and snake-like robots have an inherent grasping limitation that prevents their use in ascending sustained near-vertical or descending sustained past-vertical natural slopes. Existing legged robotic systems do not have this limitation, but still have bypassed the issue of maintaining contact with the slope by using rappel tethers. Reliance on these tethers prohibits initial cliff ascent, and limits the slope grade on cliff descent to below 90 degrees.

A wide variety of robots capable of climbing vertical *artificial* surfaces is available. Most of these robots exploit some property of the surface for easy grasping. For example, some of these robots use suction cups or permanent magnets to avoid slipping [5-8]. Others take advantage of features such as balcony handrails [9] or poles [10]. However, the surface properties that are exploited by these robots generally are not available in natural terrain.

In contrast, the simpler hardware designs used by [2, 11] had no such limitations. It is expected that solutions to the planning problem such as the one presented in this paper will allow basic *natural* vertical terrain to be climbed by similar systems, in addition to the ducts and pipes climbed by existing systems, and will suggest design modifications for better performance.

Future studies could address the use of other types of tools for grasping vertical natural surfaces, such as tools for drilling bolts or placing other types of gear in rock. The use of these tools would allow more challenging climbs to be accomplished, in the same way that "aid" helps human climbers [18, 19]. However, these tools bring an increase in weight and complexity, slowing movement and limiting potential applications.

3.2 Control

There are three primary components of the control problem for a climbing robot: maintenance of equilibrium, endpoint slip control, and endpoint force control. These three components are tightly related. In order to maintain balance, both the location of the center of mass of the robot and the forces from contacts with natural features must be controlled. Control of slip at these contacts is directly related to the direction and magnitude of the contact forces.

Existing control techniques such as those based on the operational space formulation [20] could form a baseline approach to the design of a control architecture for a climbing robot. However, these techniques could be extended in a number of different ways to achieve better performance. For example, future research might address the design of an endpoint slip controller that is stable with respect to the curvature of a contact surface, rather than with respect to a point contact only.

3.3 Sensing

For control and grasping, the robot must be capable of sensing the orientation of its body with respect to the gravity vector, the location of its center of mass, the relative location of contact surfaces from its limb endpoints, and the forces that it is exerting at contacts with natural features. For planning, the robot must additionally be able to locate new holds and generate a description of their properties, possibly requiring a measurement of levels of slip at contact points. Sensor integration, in order to acquire and use this information with algorithms for control, grasping, and planning, is a challenging problem.

Existing engineering solutions are available which can lead to the development of a baseline approach in each case. For example, sensors such as those described in [21, 22] can provide basic endpoint force and slip measurements, an inertial unit and magnetic compass can provide position information, an on-board vision system can provide a rough characterization of hold locations and properties, and encoders can provide the location of the center of mass. However, the improvement of each of these sensors—in terms of performance, mass reduction, or cost reduction—presents an open area for research.

Although the performance of the planning framework that will be presented in Section 4 would be improved with better sensor information, it does not depend on a perfect model of the environment a priori. Since the framework leads to fast, online implementation, plans can be updated to incorporate new sensor information as it becomes available.

3.4 Grasping

The performance of a climbing robot is dependent on its ability to grasp "holds," or features on a steep natural surface. It has already been noted that specialized grasping schemes, relying on specific properties of the surface such as very smooth textures, pegs, or handles, cannot be used for grasping arbitrary natural features. The problems involved in grasping natural holds will be examined further in this section.

Traditionally grasp research has been interested in either picking up an object or holding it immobile (also called "fixturing.") Research in this subject dates as far back as 1876 it was shown that a planar object could be immobilized using a minimum of four frictionless point constraints [23]. Good overviews of more recent work can be found in [24, 25]. In this field an important concept is "force-closure," defined as a grasp that "can resist all object motions provided that the end-effector can apply sufficiently large forces at the unilateral contacts." [25] Nearly all research on grasps has focused on selecting, characterizing, and optimizing grasps that have the property of force-closure.

However, for the task of climbing a grasp need not achieve force-closure to be a useful grasp. For example, a robot may find a shelf-like hold very effective for pulling itself up, even though this grasp would be completely unable to resist forces exerted in other directions. For this reason, the techniques for

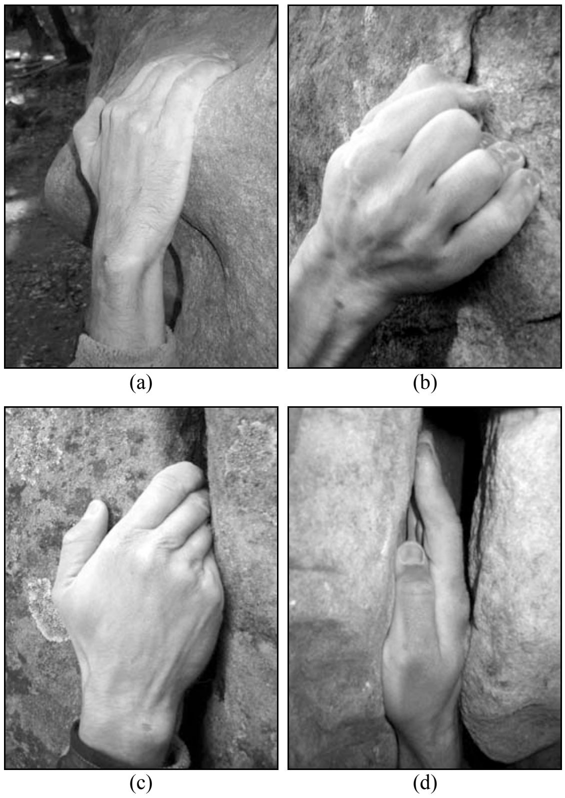

Fig. 2. Four different human climbing grasps, the (a) open grip, (b) crimp, (c) finger-lock, and (d) hand jam.

selecting, characterizing, and optimizing grasps must be expanded significantly to apply to climbing robots.

Characterization involves examining the direction and magnitudes of forces and torques (also called wrenches) that can be exerted by the grasp. For example, for one-finger grasps on point holds, an adequate representation of this information is a friction cone, which will be used for the planning algorithm described in Section 4.

The idea of characterization also encompasses a "quality factor." Measures of grasp quality have been researched extensively and are well reviewed in [26]. This work lists eight dexterity measures that include minimization of joint angle deviations and maximization of the smallest singular value of the grasp matrix. Other relevant research has been done using the concept of the wrench space. Using this concept, quality is defined as the largest wrench space ball that can fit within the unit grasp wrench space [27]. The volume of the grasp wrench space, or of more specialized task ellipsoids, could be used as a quality measure [28]. These ideas have been expanded to include limiting maximum contact force and applied in a grasp simulator to compute optimal grasps with various hands in 3D [29, 30].

However, the concept of grasp quality is ill defined for grasps that do not provide force-closure. Depending on the direction that a climber wishes to go, different grasps may be of higher quality. Furthermore, grasp quality generally includes a concept of security or stability, and this too is ill defined for non-force-closure grasps. Again, depending on the direction of applied forces, the security of a grasp may change. The concept of hold quality must be defined before useful optimization is possible. Also, an efficient way of transmitting this information to a controller or planner is necessary to accomplish the climbing task.

A qualitative classification of different types of grasps already exists in the literature for human climbers [19, 31]. In this classification, grasps are first broken into two categories, those meant for pockets, edges, and other imperfections on otherwise unbroken vertical rock faces, and those meant for sustained vertical cracks. Several examples of different face and crack grasps are shown in Figure 2. The literature gives a rough idea of the quality and use of each type of grasp in terms of criteria such as a perceived level of security, the amount of torque that can be exerted on a hold, and the amount of friction at the "power point."

Not only is this expert intuition qualitative, but also it is clear that human climbers need to perform additional grasp planning for specific cases. As put by Long, "There are as many different kinds of holds as there are ways to grab them [31]." However, this intuition can be used as a starting point for determining meaningful *quantitative* criteria for grasp selection and optimization.

A comparison of the climbing literature with past work on robotic grasp planning reveals several other fundamental differences between the two applications that may become important in future research. For example, many climbing holds are very small, so the fingers used in a climbing grasp often have large diameters relative to the object to be grasped. Literature on robotic grasping primarily considers the case where the fingers have small diameters relative to the object. In addition, some climbing grasps, as mentioned above and shown in Figure 2, are based on jamming fingers in a crack. This technique is very different from one a robot might use to pick up an object, and requires a high degree of flexibility and small degrees-of-freedom in order to "un-jam" the fingers. Clearly, continued work on climbing robots eventually will lead to the consideration of a wealth of new issues in grasping.

4 Planning

The planning problem is the fifth fundamental challenge for climbing robots in natural terrain. Details of the motion-planning framework presented in this section are given in [32].

4.1 Challenges

The planning problem for a climbing robot consists of generating a trajectory that moves the robot through a vertical environment while maintaining equilibrium.

This problem is challenging even for human climbers! Climbing is described by Long as a "singular

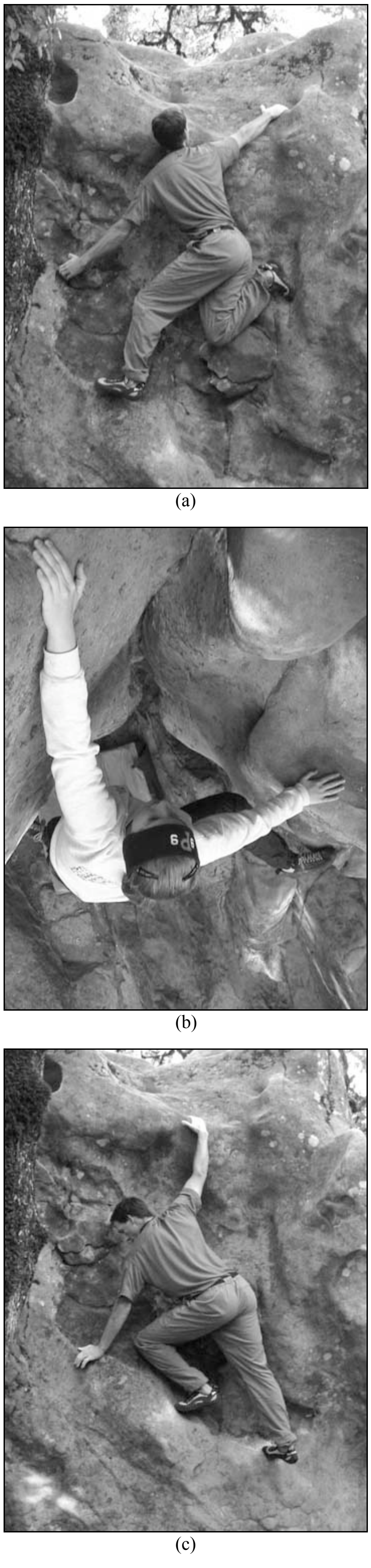

Fig. 3. Three different human climbing "moves," the (a) back-step, (b) stem, and (c) high-step.

challenge, where each 'route' up the rock is a mental and physical problem-solving design whose sequence and solution are unique. Every climb is different [31]." Much of the sequence for a particular route might be composed of one of a variety of different types of "moves," such as a back-step, stem, mantel, high-step, counterbalance, counterforce, lie-back, down-pressure, or under-cling. Some of these moves are shown in Figure 3. Each "move" is a learned technique for maintaining balance that may seem counterintuitive. In addition to these heuristics, movement through a large number of other very specific body positions might be necessary to progress towards the top of a climb.

The importance of planning a sequence of moves before actually climbing is emphasized by Graydon and Hanson [19], who recommend that climbers "identify and examine difficult sections before [they] get to them, make a plan, and then move through them quickly." The human motivation for this approach is primarily to minimize the effort required for each move and to conserve energy, since most people have hard strength and endurance limits.

The planning problem for a climbing robot is quite similar. The robot likely will be equipped with actuators that can exert high torques only for short amounts of time, so planning a sequence of moves before climbing is important for a robotic system as well. Likewise, a climbing robot will be subject to the same hard equilibrium constraints, and will need to select between a similarly wide range of possible motions. Therefore, the development of a planning algorithm for an autonomous climbing robot is a very challenging problem.

4.2 Related Work

The search space for a climbing robot is a hybrid space, involving both continuous and discrete actions. Many different methods are available for motion planning through continuous spaces, including cell decomposition, potential field, and roadmap algorithms [33]. Discrete actions can be included in these methods directly, for example at the level of node expansion in roadmap algorithms, but this approach generally leads to a slow implementation that is specific to a particular system.

Previous work on motion planning for legged robots has developed tools for addressing these hybrid search spaces for some systems. This work can be categorized by whether or not the planning is done offline, in order to generate a reactive gait, or online, in order to allow non-gaited motion specific to a sensed environment.

Gaited planners generate a predefined walking pattern offline, assuming a fairly regular environment. This pattern is used with a set of heuristics or behaviors to control the robot online based on current sensor input. Gaited planning was used by [2, 11], for example, to design patterns for climbing pipes and ducts. Other methods such as [34] are based on the notion of support triangles for maintaining equilibrium. Stability criteria such as the zero-moment-point have been used to design optimal walking gaits [35]. Dynamic gaiting and bounding also have been demonstrated [36-38]. Recent work [39, 40] has attempted to provide unifying mathematical tools for gait generation. Each of these planning algorithms would be very effective in portions of a natural climbing environment with a sustained feature such as a long vertical crack of nearly uniform width. However, something more is needed for irregular environments such as the one studied in this paper, where the surfaces on which the robot climbs are angled and placed arbitrarily.

Non-gaited planners use sensed information about the environment to create feasible motion plans online. Most previous work on non-gaited motion planning for legged robots has focused on a particular system model, the spider robot. The limbs of a spider robot are assumed to be massless, which leads to elegant representations of their free space for quasi-static motion based on support triangles [41-43]. These methods have been extended to planning dynamic motions over rough terrain [44, 45]. The analysis used in these methods breaks down, however, when considering robots that do not satisfy the spider-robot assumption. For example, additional techniques were necessary in [46, 47] to plan non-gaited walking motions for humanoids, which clearly do not satisfy this assumption. To address the high number of degrees of freedom and the high branching factor of the discrete search through possible footsteps, these techniques were based on heuristic discretization and search algorithms. This paper considers a robot with fewer degrees of freedom in a more structured search space where it is possible to achieve much better performance than with these heuristic methods. Similar issues were addressed by [48] in designing a motion-planning algorithm for character animation, although this algorithm was meant to create "realistic," rather than strictly feasible, motion.

There is also some similarity between non-gaited motion planning for legged locomotion and for grasping and robotic manipulation, particularly in the concept of a manipulation graph [24, 49-51]. Both types of planning require making discrete and continuous choices.

None of these existing planning techniques is sufficient to address even the simplest version of the climbing problem in natural vertical environments, in which quasi-static motion, perfect information, and one-finger grasps on point holds are assumed. The problem becomes even more complicated if the quasi-static and perfect information assumptions are relaxed, and if more complicated grasps are considered.

4.3 Planning Framework

In this section, we will describe our planning framework in the context of a specific climbing robot, shown in Figure 1. This robot consists of three limbs. Each limb has two joints, one located at the center of the robot (called the pelvis) and one at the midpoint of the limb. Motion is assumed to be quasi-static (as is usually the case in human climbing) and to occur in a vertical plane, with gravity. The low complexity of this robot's kinematics makes it suitable for studying the planning of climbing motions.

The terrain is modeled as a vertical plane to which is attached a collection of small, angled, flat surfaces, called "holds," that are arbitrarily distributed. The endpoint of each robot limb can push or pull at a single point on each hold, exploiting friction to avoid sliding.

A climbing motion of the robot consists of successive steps. Between any two consecutive steps, all three limb endpoints achieve contact with distinct holds. During each step, one limb moves from one hold to another, while the other two endpoints remain fixed. The robot can use the degrees of freedom in the linkage formed by the corresponding two limbs to maintain quasi-static equilibrium and to avoid sliding on either of the two supporting holds. In addition, during a step, the torque at any joint should not exceed the actuator limits and the limbs should not collide with one another. These constraints define the feasible subset of the configuration space of the robot in each step. A path in this subset defines a one-step motion.

The overall planning problem is the following: given a model of the terrain, an initial robot configuration where it rests on a pair of holds, and a goal hold, generate a series of one-step motions that will allow the robot to move in quasi-static equilibrium from the initial configuration to an end configuration where one limb endpoint is in contact with the goal hold.

In [32] we presented the details of a framework to address this planning problem. This framework can be summarized as follows.

First, we presented a detailed analysis of one-step motion for the three-limbed climbing robot. The properties of the continuous configurations at which the robot is in equilibrium were established, and were used to define the feasible set of robot configurations at each pair of holds. In particular, it was shown that the connectivity of the four-dimensional continuous feasible space of the robot could be preserved when planning in a two-dimensional subspace. This result reduced the complexity of the one-step planning problem and led to a fast, online implementation.

Then, the overall planner combined this "local planner" with a heuristic search technique to determine a sequence of holds from the initial configuration to the goal hold. The heuristic methods were based on observation of the way in which human climbers plan their motion.

4.4 Results

Our work in [32] presented only one set of simulation results, for a particular vertical environment. This paper presents a second set of results, for a more challenging environment. This environment, as shown in Figure 4, contains 50 arbitrarily placed and angled holds. The robot is initially located on the two holds at the bottom of the environment, and is required to reach the top two holds.

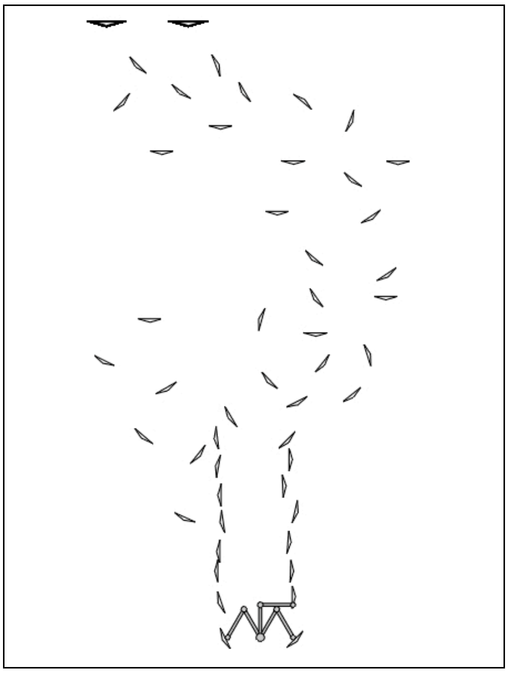

Fig. 4. An example vertical environment for the three-limbed climbing robot.

A plan was found in 3.0 seconds using a 450 MHz PowerPC processor, which is typical for an environment containing 50 holds. Planning times for smaller environments are on the order of 0.1 seconds.

A representative continuous configuration from each one-step motion in the planned sequence is shown in Figure 5. Many of these configurations are remarkably similar to human configurations. For example, the configuration shown in Figure 5(a) is similar to the "stem" shown in Figure 3(b). Likewise, Figures 5(i) and 5(n) depict configurations similar to the "backstep" of Figure 3(a) and the "high-step" of Figure 3(c), respectively.

Each frame of Figure 5 also shows the *equilibrium region* for the current pair of holds on which the robot is standing. This is the region over which the center of mass of the robot can move while remaining in quasi-static equilibrium without slipping, and is a complete specification of the equilibrium constraint on the robot. Notice that in each configuration shown, the center of mass of the robot lies within the equilibrium region, as expected.

More results, including animated 3D-visualizations, are available online at http://arl.stanford.edu/~tbretl/.

5 Conclusion

This paper described the challenges to developing an autonomous climbing robot and presented a framework for addressing the planning problem.

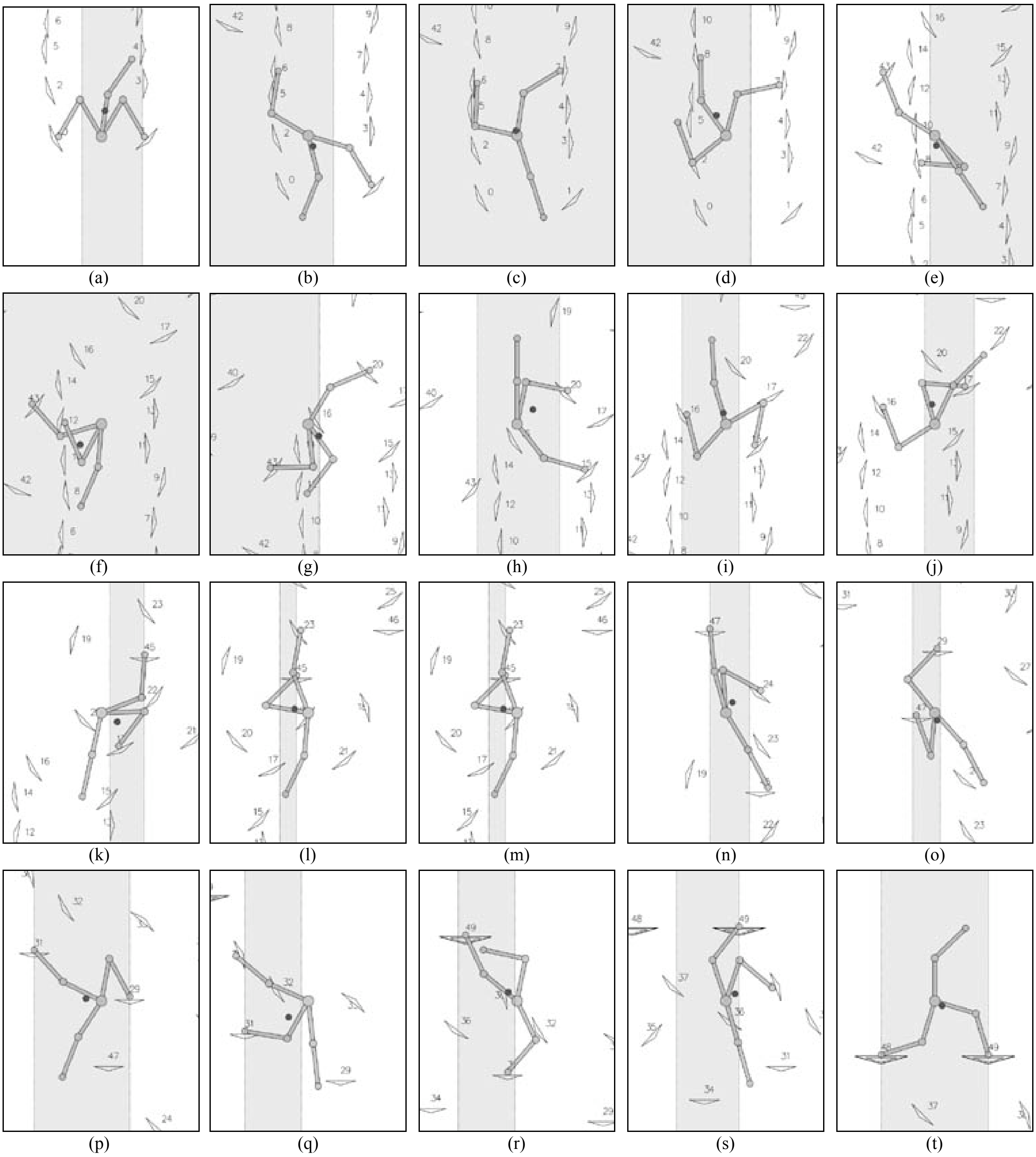

Fig. 5. Representative steps of the robot's motion for the example environment shown in Figure 4. The dark circle in each frame is the center of mass of the robot. The shaded column is the region over which the center of mass can move while the robot remains in equilibrium.

Current work deals with the application of the planning framework to a real robotic system, using real hardware. As part of this effort, the framework is being extended to handle additional motion constraints, more complicated robot geometries, imperfectly known environments, and three-dimensional terrain.

Future work will address the other four fundamental issues—hardware design, control, sensing, and grasping—and their relationship to the planning problem.

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