Modeling Human-Body Soft Tissues for Surgical Applications

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Principal Investigator

Jean-Claude Latombe

Robotics Laboratory
Computer Science Department
Stanford University
Stanford, California 94305

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1 Project Summary

The goal of this research is to develop a general-purpose computational scheme for representing the geometry and physics of human-body soft-tissue structures and simulating their displacements and deformations in real-time when external forces are applied by surgical instruments. The ability to realistically model soft tissues on a computer has the potential to radically change the ways surgery is currently taught and practiced. Virtual environments simulating deformable tissues will allow medical students to extensively practice delicate operations and confirmed surgeons to re-train themselves and maintain their skills for infrequent operations. Soft-tissue simulations will become essential to plan and rehearse surgical operations. The on-line availability of the models will make it possible to give intra-operative assistance to the surgeons. It will enable the introduction of roboticized instruments that will perform operations, either automatically or in interaction with surgeons. Research on soft-tissue modeling will directly benefit domains such as robotics, where the modeling of environments containing deformable objects is increasingly important; computer vision, where the task of tracking objects undergoing continuous deformations is central to understanding image sequences; and computer graphics, especially for the animation of virtual characters.

Previous research on soft-tissue modeling provides foundations on which the proposed research is based. In particular, as several researchers have suggested before, tissue structures will be modeled by meshes of point masses connected by elastic links, the so-called mass-spring elastic meshes. The research program includes both the theoretical development of a computational scheme based on the elastic mesh concept and the implementation/experimentation/evaluation of this scheme within a state-of-the-art virtual reality environment. The experiments will be conducted in the context of two applications: micro-anastomosis (the suturing of small blood vessels) and craniofacial surgery. This experimental effort will be critical to the validation of the results.

The unique features that this research expects to achieve are the following:

- *Modeling of complex tissue structures.* Most existing models can only represent a tissue with uniform and isotropic elastic properties. Techniques will be developed to specifically model multiple adjacent soft tissues and their attachment to bones.

- *Realistic modeling of the mechanical response of individual tissues.* To simplify computations a number of models assume linear elastic models. But human-body tissues are nonlinear and exhibit some hysteresis. These properties will be captured in the new modeling scheme.

- *Modeling changes in tissue topology.* External forces may cause topological changes in tissue structures, such as tissue cutting or tearing and formation of new contacts. A significant portion of the effort will be directed toward the effective modeling of such changes.

- *Dynamic simulation.* A simulator may only compute static equilibrium states of a tissue. Instead, by taking inertia and viscosity into account, it may compute the successive shapes of the tissue as it deforms toward equilibrium. The research will focus on the second type of simulation.

- *Automatic construction of tissue models.* Techniques will be developed to automatically generate complex elastic meshes and adjust their mechanical parameters from various imaging sources (*e.g.*, video, laser scans, Computed Tomography, Magnetic Resonance Imaging).

The research will be conducted in close collaboration between the Computer Science Department, the Functional Restoration Department, and the Stanford-NASA National Biocomputation Center. A key mission of this center is to accelerate the transfer of imaging and visualization technologies to operating rooms and surgical training.

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2 Project Description

2.1 Preamble

Over the past decade there has been considerable progress in medical imagery for diagnosis and surgical planning. Combined with advances of similar magnitude in computer graphics and geometric modeling, this progress now makes it possible to both represent complex tissue structures and perform virtual fly-through operations. Haptic transducers and 6-D trackers are permitting new ways of simulating physical interaction with modeled tissues. However, in most existing systems, simulated tissues are not deformable, which considerably restricts the kind of experience a user can get. An important goal is now to develop models for efficiently computing the displacements and deformations of soft tissues under external forces, such as those applied by surgical tools when they probe, grasp, pinch, cut, pierce, and suture the simulated tissues. Research has recently been conducted toward this goal, but more progress is critically needed before models of human-body soft tissues can widely be used to create realistic simulators.

Research on soft-tissue modeling has the potential to radically change the ways surgery is currently taught and practiced. For example, surgical training currently requires a significant number of years to complete. A trainee initially learns the anatomy from cadaver dissections, which are expensive to perform. In microsurgery, animals are used for a lengthy six-month course before the trainee can participate in actual patient procedures. The actual learning happens in the operating room, at first by assisting on simple procedures and gradually participating in more complex procedures. Over a period of many years the trainee encounters an adequate number of common problems as well as a number of unusual problems. Having a realistic computer training tool for these procedures, based on a virtual environment simulating deformable tissues in real time, would enable more effective learning and broader exposure to all aspects of the surgical specialty. In addition, this technology would allow surgical skills to be maintained. Surgeons may even be required to go through a surgical simulator for recertification, just like flight simulators are essential today for maintaining and testing the pilot's skills.

Virtual environments with realistic soft-tissue simulation will become essential to plan and rehearse surgical operations. In reconstructive surgery, for example, rearranging adjacent tissues in order to cover a defect depends on the elasticity of these tissues, as well as on their underlying fibrous attachments to bones. At times the adjacent tissues are not adequate to cover the defect; tissues from another part of the body are then used and their blood supply is connected to vessels near the defect using a microscope. Experience allows the surgeon to make the appropriate incisions and usually correct the defect. In many cases, however, a better option would have been available if the surgeon had the luxury of "practicing" ahead of time on a realistic, deformable computer model.

The on-line availability of tissue models will also make it possible to provide intra-operative assistance to the surgeons, for instance by helping them spare critical structures during a dissection. It will also facilitate the introduction of roboticized instruments that will perform operations, either automatically or in interaction with possibly remote surgeons.

The PI (Jean-Claude Latombe) has already done significant research on applying motion planning and geometric modeling techniques to medical surgery. In cooperation with Dr. J. Adler in the Neurosurgery Department at Stanford, he led the development of a treatment planner for a new stereotaxic brain surgery system [80, 89]. This system is now in clinical use at Stanford. The planner is also part of the software package sold by Accuray, Inc. Regarding deformable structures, the PI and co-workers have developed software to reason about the possible motions of flexible ligands (small molecules) to determine if they are likely to bind at targeted receptor sites [28, 29, 54, 81]. Initiated with Pfizer Pharmaceuticals, this work is now being pursued with Prof. D. Brutlag in the Biochemistry Department at Stanford. Recently, the PI has started investigating the modeling of small blood vessels [84], with Dr. M. Stephanides (Functional Restoration Department) and Dr. K. Montgomery (Stanford-NASA Biocomputation Center). The research proposed here will extend this preliminary work. Beyond this project, the PI plans to investigate motion planning techniques to guide surgical tools along optimal trajectories among soft tissues.

This project will be a collaboration between Stanford's Computer Science and Functional Restoration Departments and Biocomputation Center. This collaboration is not only essential to the successful outcome of the research, it will also allow us to quickly make our results available to medical students and surgeons. A key mission of the Biocomputation Center[1] is precisely to accelerate this kind of transfer.

We expect our research to directly benefit domains such as robotics, where the modeling of environments containing deformable objects is increasingly important; computer vision, where the task of tracking objects undergoing continuous deformations is central to understanding image sequences; and computer graphics, especially for the animation of virtual characters.

2.2 Overview of Proposed Research

Our goal is to develop a *general-purpose* scheme for representing the geometry and physics of soft-tissue structures and simulating their displacements and deformations in real time when external forces are applied by surgical tools. By "general-purpose," we mean that we will strive to develop a scheme applicable to a wide range of human-body tissues. We also expect that the scheme will be applicable to surgical training, pre-operative planning and rehearsal, as well as intra-operative assistance to surgeons and guidance of robots. In training, it will typically be augmented by software tools to evaluate the performance of the users and give them feedback. In planning, it will be augmented with motion planning techniques to compute trajectories of the surgical tools.

Our research program includes both the development of this scheme (Section 2.4) and its implementation, experimentation and evaluation within a state-of-the-art virtual reality environment, with 3-D graphics, haptic devices, and object trackers provided by the Biocomputation Center (Section 2.5). In association with the Functional Restoration Department, we will conduct our experiments in the context of two specific applications: *micro-anastomosis* (suturing of small blood vessels) and *craniofacial surgery*. These applications will guide our technical choices to design an appropriate soft-tissue modeling scheme and will be used to evaluate the scheme and validate our results. We consider this experimental work to be a major component of our project and we expect to devote at least one half of our time to it.

There has already been substantial research on soft-tissue modeling (Section 2.3). Despite serious shortcomings, the current results provide the foundations on which we will develop our own research program. In particular, as several authors have done before us, we will explore the modeling of tissues by meshes of point masses connected by elastic links, the so-called mass-spring elastic meshes. We stress below some unique features that we hope to achieve:

• *Modeling of complex tissue structures.* Most existing models only represent tissues with uniform and isotropic elastic properties. We will investigate techniques to represent multiple connected tissues, such as skin, fat, muscles, tendons, vessels, and their attachment to bones.

• *Realistic modeling of the mechanical response of individual tissues.* To simplify computations a number of models assume linear elastic models. However, most human-body tissues are nonlinear and exhibit some hysteresis. We wish to capture these properties in our design.

[1] URL: http://www-biocomp.stanford.edu/

• *Modeling changes in tissue topology.* External forces may cause topological changes in tissue structures, such as tissue cutting or tearing, formation of new contacts, and gluing of contacts. A significant portion of our effort will be directed toward the modeling of such changes.

• *Dynamic simulation of tissue deformations.* A tissue simulator may ignore inertia and viscosity, by only computing equilibrium states; or it may take these parameters into account, and compute the successive shapes of a tissue as it deforms toward equilibrium, making it possible to animate the tissue in real time. We will focus our research on the second type of simulation.

• *Automatic construction of tissue models.* Models of complex tissues cannot be constructed by hand. We will develop techniques to automatically generate elastic meshes and adjust their mechanical parameters from various imaging sources (video, laser scans, CT[2], MRI[3]).

Future simulators for surgical applications will have to take the functional nature of organs into account, in addition to the elastic properties of the tissues. For example, cutting a blood vessel locally reduces blood pressure, which may in turn cause forces between tissues to vary [23]. Processes like breathing may also affect tissue deformation. However, the research that we propose here will not consider such interactions; we will leave them for future work.

2.3 Review of Prior and Current Work

2.3.1 Background

Medicine and surgery offer a broad application field for virtual reality technology. This is attested by the recent flurry of conferences and journals on this topic and by the creation of new research centers (including [6, 14, 40, 44, 65, 84]). Most existing medical simulators use anatomic geometric representations and ignore the elastic properties of tissues. As the human body mainly consists of soft tissues, the development of realistic soft-tissue models will not only result in much better simulators, it will also make these simulators more broadly applicable. Therefore, it is not surprising that this area has recently attracted much attention, *e.g.*, [2, 6, 8, 10, 11, 12, 19, 20, 21, 31, 34, 49, 51, 52, 53, 59, 60, 61, 62, 64, 69, 72, 78, 82, 93]. Excellent reviews can be found in [26, 32, 33].

The core problem is the following: Given a tissue at a resting position, compute its displacement and deformation when external forces are applied by surgical instruments. When a force-feedback device is used to interact with the tissue model, one must instead compute the reaction force exerted by the tissue on the tool given the tool's position. In a *static* simulation system, only the equilibrium state of the tissue is computed; inertia and viscosity are ignored. In *dynamic* simulation, the changes in the shape of the tissue as it moves toward equilibrium are computed and displayed; then the dynamic laws of motion must be taken into account. Most medical applications require dynamic simulation; for others, static simulation is sufficient.

Experimental data have been collected in Biomechanics about human-body tissue elasticity (*e.g.*, [30, 50]). These studies show that tissues do not have a linear response: the curve representing the stretch (strain) of a tissue as a function of the applied force (stress) is typically a J-shaped curve; as the tissue gets closer to tearing, the increase in stretching becomes smaller per additional unit of exerted force. Moreover, the tissue response exhibits hysteresis: the curves for increasing and decreasing forces are different. Studies also show that most tissue structures are non-homogeneous (*i.e.*, the stress-strain curve varies across the structure) and anisotropic (the curve depends on the direction of the applied stress). Finally, most tissues are viscous, a property that can be modelled by introducing a force proportional to the deformation rate in the equations of motion.

[2]Computed Tomography.
[3]Magnetic Resonance Imaging

Biomechanical models provide data needed to build/evaluate realistic simulators, but do not tell us how to design computational models. The two most critical issues in designing such models are *accuracy* — the simulator should generate visual and haptic sensations that are very close to reality — and *efficiency* — deformations must be rendered in real time on the graphic display and the haptic device.

2.3.2 Approaches to soft-tissue modeling

Virtually all soft-tissue models proposed so far are based on the same general principle: the tissue is represented by a mesh structure through which deformations caused by external forces are propagated by iterating a basic computation. However, there are important differences between the models, which impact simulation realism and computational complexity. The two most widely proposed variants are finite-element methods (FEM) and mass-spring elastic meshes.

FINITE-ELEMENT METHODS [2, 7, 19, 34, 60, 78]: Let Ω be the domain occupied by a tissue before deformation. This domain is decomposed into a mesh of carefully selected elements, for instance, tetrahedra. Using variational principles to minimize the potential energy of the tissue [32], one gets a system of the form:

$$Ku = F$$

where K is the stiffness matrix, u is the unknown displacement field (*i.e.*, the vector field representing the displacement of every tissue point), and F is the vector of internal and external forces applied to the tissue. The FEM computes u over each mesh element as a combination of preselected functions. Ω, hence the mesh, remains fixed during the computation despite the fact that the domain occupied by the tissue changes. For dynamic simulation, the equations of motion yield a system of the form:

$$M\ddot{u} + C\dot{u} + Ku = F$$

where $\dot{u}$ and $\ddot{u}$ are respectively the first and second derivatives of u relative to time, M is the inertia matrix, and C the damping matrix modeling tissue viscosity. The computation of the matrices M, C, and K and of the vector F, which involves numerical integration over Ω, is computationally expensive [31]. However, it is usually done in a preprocessing phase.

MASS-SPRING ELASTIC MESH [6, 25, 31, 34, 51, 93]: A mass-spring system consists of a mesh of point masses connected by elastic links. Here, the mesh represents the tissue, not the domain it initially occupies. Each node of the mesh is attached to a given point of the tissue. Thus, when the tissue is deformed, the mesh deforms with it.

The mesh is used to discretize the equations of motion. The links connecting pairs of mesh nodes allow the local description of the elastic properties of the tissue. Any change of length relative to a resting length causes an internal force to occur between the two connected nodes. To dynamically simulate tissue deformation one models the effect of inertial and damping forces with a system of second-order differential equations. Each node N in the mesh is subject to an equation of the form:

$$m\ddot{p} + c\dot{p} + \sum_i F_i = F_{ext} \tag{1}$$

where p is the coordinate vector of N, $\dot{p}$ and $\ddot{p}$ are its velocity and acceleration, respectively, m is the mass attached to N, c is the damping coefficient modeling viscosity, F_i is the internal force exerted by a node N_i to which N is connected by a link (the sum $\sum_i$ is taken over all such nodes

N_i), and F_{ext} is the total external force applied to N (force exerted by a surgical tool, gravity). The force F_i is defined by:

$$F_i = k_i(\|p - p_i\| - \ell_i)\frac{p - p_i}{\|p - p_i\|}$$

where p_i is the coordinate vector of N_i, k_i is the stiffness of the link connecting N and N_i (k_i may not be constant), and ℓ_i is the resting length of this link. The system of equations obtained for all the nodes in the mesh is solved through time using numerical techniques.

2.3.3 Reduction of computational time

When applied to complex tissues, both FEM and elastic mesh methods yield computationally demanding systems of equations. Techniques have been proposed to reduce computational time.

One approach is to *simplify equations*. This can be done by only allowing static simulation (*e.g.*, [19, 26]). Since it does not require time integration, static simulation is potentially faster than dynamic simulation. Another way of simplifying equations is to assume linear elasticity. In [11, 20] this assumption results in speedups of 100, or more. Another way of simplifying equations is to combine kinematic and dynamic models. An example is the elastic mesh model called ChainMail presented in [31]. Kinematic inequality constraints determine the coarse motion of the mesh and a dynamic mass-spring model is used to refine the final shape of the object.

Another common technique to reduce computational time is to *reduce the size of the mesh*. For example, in [12] the system of equations is simplified by solving for displacements only in surface elements and by assuming a small number of externally applied forces. In general, one could consider surface meshes, rather than the more expensive volume meshes, but surface models seem inherently unable to capture the behavior of complex tissue structures. In [78] the spherical shape of the tissue of interest (the cornea) is exploited to create a relatively coarse mesh. Mesh density can also be adapted to the precision needed. Tissue regions of less interest are then modeled by meshes of lesser density.

A third way to achieve faster simulation is to perform *preprocessing operations*. For instance, in [11, 71] the deformation modes of a given object are precomputed; then the deformation for an arbitrary force is computed by superposing several deformation modes. Superposition, however, usually assumes a linear model of tissue elasticity, which may not always yield realistic results. Moreover, because preprocessing takes several hours, changes in tissue topology cannot be considered [19].

2.3.4 Force feedback

The sensation of forces is very important in virtual environments for surgical applications [5, 43, 58]. Both FEM and elastic mesh methods allow the computation of forces. In [2] the motion entered by the surgeon on the haptic device is used to compute tissue deformation. The force at the point of contact between the tip of the surgical instrument and the tissue is derived from the internal forces computed in the tissue. The surgeon-tissue loop is closed by rendering this force through mechanical actuators on the haptic device.

It is important to note that, to be realistic, forces must be rendered on the haptic device at a high frequency. They must also be carefully synchronized with graphic animation [26]. It is desirable that graphic rendering be done at video rate, *i.e.*, 30 Hz; for force feedback, the rendering frequency should range between 300 and 1000 Hz. This means that accurate forces and deformations should be computed at least 10-30 times a second, so that faster extrapolation techniques can reasonably be applied during each time step.

2.3.5 Medical applications of soft-tissue models

<u>ANASTOMOSIS:</u> Anastomosis, the task of suturing tube-like organs, is ubiquitous in surgery. It applies to blood vessels, ureter, trachea, colon, bile duct and other organ systems. An integrated system that allows users to perform a simulated end-to-end anastomosis using virtual tweezers, needle-holder, needle and thread is described in [8]. The tube models deform when grasped by the virtual tools, which are attached to force-feedback devices. This system includes software that measures critical parameters of the user's performance.

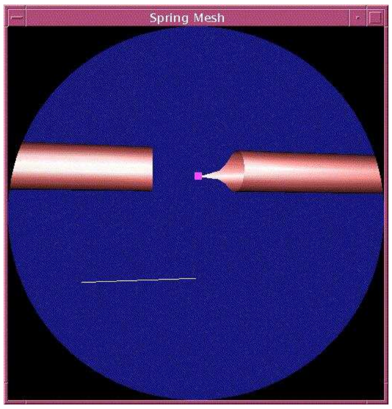

Figure 1: Virtual anastomosis

In [60] an elastic tubular surface tissue is modeled using a FEM technique. In [84] we describe preliminary results using a mass-spring elastic mesh to model thin blood vessels and simulate microvascular suturing operations. Real microvascular operations are done on vessels that are too small to return perceptible forces; the surgeon, who observes the vessels through a microscope, must entirely relies on his/her visual sense. Our system provides a virtual graphic environment similar to the microscope environment; it computes in real time the deformations of the vessel and simulates the surgical tools allowing the surgeon to perform suturing operations.

<u>CRANIOFACIAL OPERATIONS:</u> Craniofacial operations are complex operations usually involving multiple types of tissues and bone cutting [67]. A mass-spring elastic mesh model is used in [25, 34, 93] to model soft facial tissue. In [34], a volume-preservation force is added to the internal forces in the mesh to deal with the constraint that some tissues are almost incompressible. In [51] a FEM is used to represent the skin surface, while a mass-spring model represents fat tissue; the spring stiffnesses are automatically derived from the intensity of the voxels in 3-D CT images. This model is used to predict the post-operative appearance of a patient whose bone structure has been changed.

<u>OTHER APPLICATIONS:</u> Several other applications of soft-tissue modeling to surgery have been presented in the literature. We survey some of them below.

The system in [78] presents a FEM model to represent the eye's cornea. The roughly spherical shape of the cornea naturally leads to describing the mesh in spherical polar coordinates. The material properties are assumed to be homogeneous and nonlinearly elastic with a J-shaped force-stretch behavior. On an SGI Reality Engine, the system achieves graphic update rates greater than 10Hz, but the cornea is a relatively simple tissue to model. Another FEM modeling of the cornea is reported in [73].

Simulation of hepatic surgery is described in [2, 19]. The liver is represented by a FEM model that assumes isotropic but not necessarily uniform elastic properties. It does not model internal vessels and ducts. The system relies on a preprocessing stage to reduce computations during simulation. Elementary deformations for a number of external forces are precalculated and stored. During simulation they are linearly superposed. After the preprocessing stage, deformations are computed in less than a few milliseconds. But precomputation is expensive and increases quickly with the size of the mesh. In a reported experiment where the mesh contains 193 vertices and 725 tetrahedra, precomputation takes 30min (on a 233 Mhz DEC Alpha); when the mesh size increases to 1735 vertices and 8124 tetrahedra, precomputation time jumps to 36 hours. If the complete structure of internal vessels and ducts were modeled, the mesh would have many more vertices. A non-linear elasticity model of the liver has also been developed by the same research group [20].

Surface mass-spring models have been used to represent the gall bladder in [21, 52]. In [6] a surface mass-spring model is used to simulate laparoscopic surgery in a virtual surgery training simulator. The surface of the deformable tissue is represented by a mesh of size 50×50. Realistic deformations of this surface model are reported.

Deformable models (mostly mass-spring elastic meshes) have also been used to simulate the deformation of a human thigh when an echographic probe is slided along the skin [24], to analyze the behavior of knee ligaments [24], to simulate skin deformation [53], to study cardiopulmonary interactions [49] and left ventricular wall motion [69], and to simulate trunk respiration [74].

2.3.6 Deformable models in graphics, vision, and motion planning

Techniques have been proposed in computer graphics for simulating deformable objects in non-medical domains [3, 24, 59, 62, 64, 86]. In many cases physical accuracy is less important than in medical applications. One area that has attracted considerable interest is *cloth modeling*. See [68] for a review. However, most cloth fabrics differ from human-body tissues; they strongly resist stretching motions while being very permissive in allowing bending motions. FEM techniques are used in [18, 36] and mass-spring models in [4, 9, 22, 75].

Another area that uses deformable models is *facial animation* [85, 88, 91]. In [88] a mass-spring mesh with four layers of nodes is used. The external layer represents the epidermia, while the innermost layer is attached to the bone structure. The three intermediate layers of links have different spring constants to represent the properties of the dermis, fat tissue, and muscle tissue. The model has 6500 springs and is animated at interactive rates.

FEM models [16] and mass-spring models [15] have also been used to animate muscles in human character animation. Mass-spring models have been used to visualize objects that experience state transitions from solid to liquid [87] and to animate artificial fishes and aquatic plants [90]. Particle systems controlled by potential fields have been used successfully to animate fluids, such as water, clouds, or flames, and to simulate crowds of people or flocks of animals.

Approximate continuum models that adhere less strictly to the laws of physics than FEM and mass-spring models, *e.g.*, snakes [47], have been applied in computer vision to track deformable objects in video sequences [17, 59, 63, 64, 69, 70].

Motion planning with deformable objects is investigated in [48]. The moving object is an elastic plate represented by a Bézier surface. The planner computes a trajectory of this object that avoids collision with rigid obstacles.

2.4 Areas of Investigation

Our goal is to develop a general-purpose computational scheme for simulating deformations of human-body tissues. This scheme should allow real-time and accurate simulation. It should allow the modeling of highly deformable tissues and structures made of multiple connected tissues each having different mechanical properties. Finally, it should be able to support changes in tissue topology (*e.g.*, tissue cutting).

As noted in Subsection 2.3, most models proposed so far represent human-body soft tissues by meshes through which deformations caused by external forces are propagated by iterating some computation. The two main families of models are the FEM and the elastic mesh models. Future models are most likely to be of one of these families, perhaps combining features of both.

FEM methods have been extensively studied in numerical analysis. They are often regarded as a more rigorous representation of soft-tissue physics than mass-spring models, because they do not discretize the equations of motion and they produce a solution over the entire domain [26]. However, their computational cost tend to be greater than that of mass-spring models, especially when nonlinear elastic models and dynamic simulation are considered. In addition, the mesh used by a FEM represents the domain occupied by the tissue before deformation, not the tissue itself. The tissue domain after deformation can be very different, since some human-body tissues can stretch by more than 100% [33]. This violates a basic assumption of the FEM, which was originally developed to compute functions or vector fields over fixed domains. For a similar reason, FEM methods make it difficult to deal with changes in tissue topology, barring a prohibitive computational cost. Another argument in favor of mass-spring meshes is that their meshes directly represent the local properties of tissues, hence facilitating the description of anisotropic, nonlinear, nonhomogeneous tissue structures.

It thus appears that for tissues that do not deform much, FEM probably provide a better model than mass-spring models. But for a greater variety of human-body tissue structures we believe that elastic meshes show greater promise. Hence, our research will focus on these meshes. We will specifically investigate a number of issues with mass-spring models that are still poorly resolved, as well as other important questions that have been rarely addressed before. We will also try to import ideas from FEM models into elastic meshes, when appropriate.

We now present the main areas that we plan to investigate.

1. IDENTIFIED ISSUES WITH ELASTIC MESHES: As mentioned above, a number of issues identified by previous research still need to be explored.

The *topology and density* of a mesh are key issues because they define how the continuous physics is discretized. Various topologies, such as regular, tetrahedral, and hexahedral lattices, are proposed in the literature. If there are not enough elastic links per mesh node, several equilibrium positions may be possible for the simulated tissue and the simulator may generate an incorrect one. A greater number of links per node and a denser set of nodes are likely to yield more accurate simulations, but they also increase the number of equations, hence computational time. A tradeoff is therefore necessary. Clearly, areas where the tissue is anisotropic require more links per nodes to precisely model the fact that the tissue has different responses along different directions. Areas where the tissue is not homogeneous or highly deformable must be sampled with a greater density of nodes.

The choice of appropriate *stiffness parameters* in the node equations of motion (Eq. (1)) is another issue. Most mass-spring models proposed in the literature use linear springs. Though this assumption simplifies somewhat the equation of motion at each node, it is undesirable for many tissues that are nonlinear and moreover exhibit hysteresis. We will investigate the use of non constant link stiffnesses in the motion equations. It has been recognized that when stiffness becomes large (*e.g.*, when a tissue is stretched close to tearing), the numerical integrator must take smaller time steps to ensure solution stability. But reducing time steps increases computation. A better alternative might be to locally increase mesh density in stiffer tissue areas. Stiff links may also be needed to represent thin tissues that are resistant to bending (*e.g.*, tubular organs).

The basic mass-spring elastic mesh model does not allow to enforce some *global properties*, such as the incompressibility of some tissues. One way to deal with this issue is to compute tissue volume while the motion equations are being solved and to apply additional pressure forces to mesh nodes located on the tissue surface in order to maintain constant volume. Such a technique has already been used, in particular in [34, 88].

2. Automatic construction of models: To realistically represent a human-body tissue, an elastic mesh may contain several thousand nodes. Such a mesh must be constructed automatically. There are two aspects to consider: the construction of the mesh structure and the selection of its elastic properties (stiffness and viscosity parameters in Eq. (1)).

The automatic construction of meshes has been widely studied in CAD and there exists a number of available general-purpose packages (*e.g.*, see [83, 79]). More specifically, the construction of the mesh structure can be done from medical images using automatic mesh generation algorithms [1]. For example, in [92], the multi-layer mass-spring facial model of [88] is automatically built from laser-scanned data. For most tissues, prior knowledge about their homogeneity and isotropy (or lack of such characteristics) should make it possible to choose the density and connectivity of the mesh appropriately. Mesh optimization techniques are also discussed in [42]. We expect that experimental work on various tissue structures will allow us to establish heuristic rules for constructing efficient meshes.

A more delicate problem is to select the values of elastic properties throughout the mesh. In most cases, initial values may be obtained from existing data. However, there are identified problems with such data. Many authors have recognized that quantitative biomechanical information is often lacking for modeling accurately human-body tissues. Most existing data have been obtained with uniaxial samples and thus only provide stress-strain curves along one dimension; their use for 3-D modeling is not straightforward, except for homogeneous and isotropic tissues. Moreover, the experiments to get these data have usually been performed *in vitro*, while it is known that *in vitro* tissue properties can differ from *in vivo* properties [26].

We propose to develop techniques to automatically adjust the elastic property of a mesh by comparing its deformation to the observation of the deformation of a real tissue. Various imaging modalities can be used (laser scans of external tissue surface, CT, MRI, ultrasound). We will develop software to track the deformation of a given tissue, match its current shape with the one computed by the simulator, and adjust the mesh parameters to reduce the differences between the two shapes. Two key algorithms will be the matching and the parameter-adjustment algorithms. Matching can be done using existing registration techniques (*e.g.*, [37, 55]). For parameter adjustment, some work has already been done. *E.g.*, genetic algorithms have been used to identify the spring parameters for a cloth animation model [22] and for a human thigh model [46]. In [27] spring parameters are obtained using simulated annealing. Backpropagation techniques inspired from those used in neural nets may also be applicable, especially when the imaging technology only allows the measurement of the deformation of the external tissue surface. In [51] stiffness parameters are automatically derived from the intensity of the voxels in 3-D CT images.

3. Multi-tissue structures: Surgery often has to deal with multiple types of tissues. For example, a craniofacial operation may have to deal with skin, dermis, fat, muscles, tendons, nerves, blood vessels, and bone. Each tissue has its own mechanical properties. Two tissues might be simply adjacent (*e.g.*, dermis and fat) and interact at their common boundary. But the relation is often more complex. For instance, a tissue may be traversed by blood vessels and tendons; while

the former could have homogeneous and isotropic properties, the entire structure does not.

One way to deal with multi-tissue structures is by locally adapting the density and the elastic properties of the mesh. For instance a significant blood vessel traversing fat tissue could yield a dense tubular mesh throughout an otherwise regular and relatively coarse mesh. A similar approach has already been studied in [88] for facial animation, yielding a multi-layer elastic mesh (see Subsection 2.3.6).

We believe that it would be preferable to represent each distinct tissue more explicitly, rather than blending them into a single mesh. For example, one could use a "multi-mesh" model such as the one that we sketch now. In this model, each tissue is represented by its own elastic mesh. The entire multi-tissue structure is thus a collection of meshes augmented by spatial relations specifying the surfaces in contact and possibly the contact types (*e.g.*, glued or sliding). These relations would be used to connect the meshes together and propagate deformations.

Such a representation has several potential advantages. It should simplify the modeling of a complex tissue structure since each mesh clearly represents a single tissue. It should also reduce computations by making it easier to assess the global properties of a substructure and to define "zones of influence" that can be used to limit the propagation of a computation (see paragraph 5 below). Finally, it will facilitate the explicit visualization of the bounding surface of each tissue during deformation.

4. Changes in tissue topology: During a simulated surgery, two major types of changes in tissue topology may occur:

- *Tissue breaking:* A tissue may be *cut* by a scapel or *torn* if it is stretched beyond some limit.

- *Tissue joining:* Two portions of tissue surface may make contact with one another. This contact may be temporary; it may also be glued by a suturing operation.

Tissue breaking can be simulated by removing mesh links [33]. For example, whenever a knife instrument is brought in contact with a simulated tissue, we can compute the mesh links that the instrument intersects and remove them from the current mesh. Similarly, while a tissue is being deformed, the length of every mesh link can be monitored so that whenever a link is stretched beyond some threshold, it is removed from the mesh.

Link removal is simple in its principle, but it may raise several difficulties. One is that the area where a tissue breaks is often an important one that will likely have to be modeled with greater precision. Technically, this can be done by locally increasing mesh density, which requires changing the mesh structure dynamically. But quantitative biomechanical data concerning the behavior of breaking tissue is also limited. This limitation could be alleviated by using medical imaging and extending the techniques proposed in the above paragraph 2. Another difficulty might be caused by the fact that the removal of some links can have a major dynamic effect on a large subset of the mesh, since it instantaneously suppresses internal forces. This may quickly create or reduce tensions on other links. Simulating such changes is likely to require reducing the integration time step to better track the deformation. Yet another difficulty (but a lesser one) will be to dynamically maintain an explicit representation of the external surface of the tissue.

Tissue joining raises issues of its own. It requires monitoring the self-collision of a deformable body (see paragraph 5 below). Once a new contact has been detected, self penetration must be avoided. This can be accomplished by dynamically increasing mesh density in the contact neighborhood, especially in the two portions of tissue surface that made contact. Similarly, when a contact is broken, the mesh density should be decreased to reduce the computational load. Two types of contact should be considered: sticky and sliding contacts. For sticky contacts, some authors have proposed introducing links with high stiffness. Since such links yield integration issues, we believe that a better technique is to temporarily merge mesh nodes in the contacting portions of surface. Appropriate treatment of contacts will require developing a force model describing the interaction between two deformable bodies.

5. Reduction of computation: Any relatively complex tissue structure will yield a large system of differential equations such as Eq. (1). Solving these equations through time to allow real-time graphic and haptic rendering is a major challenge. As mentioned in Subsection 2.3.3, a number of techniques have been proposed to accelerate simulation. We will use several of these techniques in combination: simplify equations by locally linearizing their parameters, reduce the size of the mesh where deformations need not be computed very precisely, and precompute typical deformations. Other techniques can be applied as well. For example, one can use adaptive [24] or larger integration time steps (as long as stability is preserved) and fast interpolation techniques between time steps to achieve the desired haptic and graphic rendering rates (a similar technique is widely used in robot control). The definition of "zones of influence" with fall-off coefficients [91] may also help focusing on a subset of the mesh. Parallelization using several PCs or workstations on a local network is also an option.

Overall the problem is mainly a resource allocation problem. Assuming that computational resources are limited and that the more precise the simulation the better, no time should be spared. But simulation needs are not the same across a given tissue structure. For example, when a surgical tool makes contact with a tissue, one would like the simulation to be very precise in the tissue surrounding the contact, in particular to generate haptic feedback very close to reality. Tissues that are far away from any external forces may be less important. Clearly, whenever mesh density is increased somewhere, some computational gain should be made somewhere else. This is analogous to what happens in Computer Graphics when fly-through navigation through a geometric model is performed with a virtual camera moving at constant velocity. To prevent local variations of the model complexity from slowing down the camera, portions of the model that are less important (*e.g.*, located far away from the camera) are rendered in less detail.

Reducing computations (as well as other events, such as changes in tissue topology) will require to dynamically and locally adapt mesh density [61]. We project to devote a specific effort to define a mesh data structure that allows such an operation at a very small computational cost. One possibility is to use a hierarchical data struture that represents the mesh at various levels of density. For example, each volume element at one level could be decomposed into smaller elements at the next level. Such a nested structure would make it possible to quickly adapt the mesh density locally by moving up and down in the hierarchy.

Another computation that will deserve attention is contact detection. Contacts may occur between a surgical tool and a tissue, or between two parts of a tissue structure (e.g., between the two ends of a blood vessel being sutured). Collision detection has attracted considerable attention in geometric modeling and robotics (*e.g.*, [41, 57]). Two approaches are used to accelerate this operation. One approximates objects by hierarchies of simple volumes (typically, spheres or bounding boxes) and uses these approximations to quickly identify the regions where a collision may occur [35, 76]. The other exploits spatial and time consistency properties of physical objects (basically, if two objects move relative to one another, their two closest features do not change often) [56]. The first approach applies well in environments made of rigid objects, because hierarchical approximations can then be precomputed. When objects are deformable, techniques to efficiently maintain dynamic data structures [39] and kinetic data structures [38] might be used instead and combined with the hierachical mesh structures proposed above. Other contact detection techniques applicable to deformable objects are proposed in [13, 45].

2.5 Experimental Work

We will implement our modeling scheme and we will test it on several examples of soft tissues. We will start our experimental work during the first year of our research (see Section 2.6). Experimental results will be used to evaluate our current modeling techniques and to guide our next research. We see this experimental work as a major component of the total project.

1. EVALUATION CITERIA: Evaluation will be done with respect to several criteria:

- *Ease of use of the modeling scheme:* We will analyze the difficulties encountered in modeling a new tissue structure or in adapting a generic model to a specific patient (*e.g.*, tuning appropriately the mesh structure and the elastic parameters).

- *Realism/accuracy of the simulations:* We will use qualitative feedback from surgeons, as well as quantitative comparisons with actual tissue deformations recorded in imaging sequences.

- *Computational efficiency:* The key issue will be to verify if simulations can be done in real time without having to greatly scale down their precision. If real time is obtained by means of precomputations, it will be important to assess the loss of flexibility that this may cause.

- *Generality of the modeling scheme:* We will assess the adequacy of the scheme to represent a variety of human-body tissue structures and to be used in a variety of contexts (training, planning, rehearsal, tele-surgery, and on-line guidance).

To the extent that comparable data will be available, we will compare our models to previously developed models.

2. EXPERIMENTAL SETUP: To conduct our experimentations, we will embed our software in the virtual reality environment available in the Biocomputation Center of the Stanford Medical School. This environment offers large-screen 3-D graphic visualization facilities, surgical tools equipped with magnetic/optical/inertial trackers to localize them in space, surgical tools mounted on force-feedback devices, and a laser scanner to acquire 3-D images.

We will use the laser scanner to effectively measure the deformations of soft tissues and to design software to automatically build elastic meshes. A force sensor will be used to compare the computed forces rendered on the haptic device and real forces applied to a tissue. Dynamic 3-D visualization of the simulated and sensed shapes will facilitate comparison.

To compare with tissue deformations *in vivo*, we will also use imaging sequences obtained during real surgery. Such sequences already exist for a number of operations, *e.g.*, blood vessel suturing. It will then be relatively easy to track tissue deformations and compare them to simulated deformations.

3. APPLICATIONS: We plan to specifically work on two applications: *micro-anastomosis* and *craniofacial surgery*. One participant in this project, Dr. Michael Stephanides, is a Plastic and Reconstructive Surgeon, who routinely performs such operations.

- *Micro-anastomosis* involves the suturing of 1mm vessels and nerves under a microscope. It is widely used for the reattachment of severed fingers, reconstructions involving skin, muscle and bone transfers from remote body areas, reversing vasectomies in urological surgery, reversing tubal ligations in gynecological surgery. The surgeon needs to place about 8 knots to join the ends of a vessel as thin as a human hair. The needle and suture are barely visible under the naked eye.

Micro-anastomosis requires the modeling of relatively simple tissues. But simulation must be both very realistic and performed in real time. Moreover, the tissue is thin along the radial dimension, which will test the ability of our model to represent thin tissue that are somewhat resistant to bending. Other complications include the fact that the tissue is neither linear nor isotropic, and the need to model both tissue piercing by a needle and contact between the two ends of the blood vessel. Both needle-tissue and tissue-tissue contact events will require locally modifying the mesh structure. Anastomosis also offers the opportunity to include a number of surgical tools in the simulation, including two forceps, a needle and a thread. For micro-anastomosis, force feedback is irrelevant; but we will try to extend our model to larger tube-like organs, for which haptic interaction can be very useful.

- *Craniofacial surgery* requires dealing with tissue structures that are considerably more complex. A good surgical outcome depends on making precise cuts in facial and skull bones and securing the soft tissues back together over the reconstructed bones. Having a tool to assist the surgeon in developing a blueprint of the entire operation and visualizing its results ahead of time would result in diminished operating time and improved outcome. However, real-time constraints are somewhat less stringent. For example, the surgeon may mostly be interested in predicting the post-operative appearance of a patient. Then precision is needed, but the computation need not be real-time. In other cases where real-time dynamic simulation is needed, the simulation often does not have to be accurate everywhere.

Craniofacial surgery will test the ability of our model to represent multi-tissue structures. This will be a formidable computational challenge for all our algorithms, especially those in charge of dynamically and locally adapting the mesh density and managing changes in tissue topology.

2.6 Research Agenda

<u>YEAR 1:</u>

- Develop a first version of our modeling scheme for representing relatively simple tissue structures.

- Investigate techniques to reduce computational time, including nested hierachical meshes.

- Develop software to construct mesh models of simple tissues from laser-scanned 3-D data.

- Demonstrate our results on microvascular anastomosis, using a needle mounted on a 6-D tracker as the surgical tool.

- Evaluate these results using 3-D visualization techniques to qualitatively assess realism and vision techniques for quantitative comparison with deformations of real blood vessels.

<u>YEAR 2:</u>

- Further develop and implement nested hierarchical meshes.

- Implement techniques for efficient contact detection among deformable objects.

- Study multi-mesh structures to represent spatially complex tissue structures.

- Start investigating dynamic mesh modifications to model changes in tissue topology.

- Demonstrate software to automatically construct elastic mesh models from imaging data.

- Demonstrate our techniques on a simple craniofacial operation.

<u>YEAR 3:</u>

- Integrate several previously developed techniques (nested meshes, contact detection, changes in tissue topology).

- Improve software to automatically construct elastic mesh models from imaging data.

- Perform more advanced demonstrations of our models to simulate craniofacial operations.
- Conduct qualitative and quantitative evaluation of our results in those demonstrations.
- Assess the successes and shortcomings of our research.

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